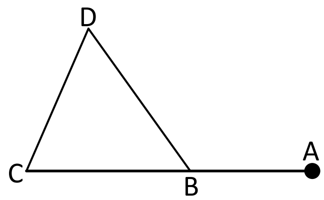
4-3 Angle Relationships in Triangles

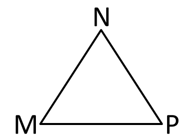
Objectives: G.CO.10: Prove theorems about triangles.

For the board: You will be able to find the measures of interior and exterior angles of triangles and apply these theorems.

Bell Work 4.3:

1. Find the m<DBA, if m<DBC = 30°, m<C = 70°, and m<D = 80°.
2. What is the complement of an angle with measure 17°?
3. How many lines can be drawn through N parallel to $\overleftrightarrow{MP}$?





Anticipatory Set:

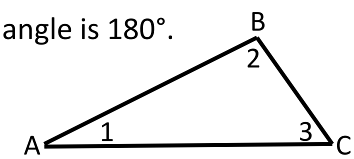
Experiment: Given a triangle, tear off the 3 corners. Arrange them so that they are adjacent angles. What do you notice about the sum of these three angles?

They make a straight angle and thus add to equal 180°

Instruction:

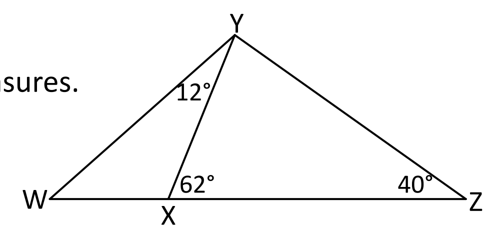
The Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.



Given: Triangle ABC
Conclusion: m<1 + m<2 + m<3 = 180°

Open the book to page 232 and read example 1.
Example: Use the diagram drawn from to find the indicated angle measures.



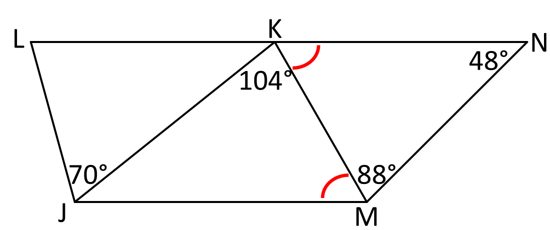
a. m<XYZ
   62 + 40 = 102
   180 – 102 = 78°

b. m<YXW
   180 – 62 = 118°

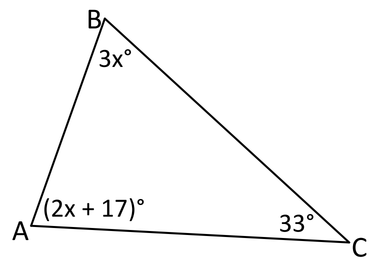
c. m<YWX
   12 + 118 = 130 **180 – 130 = 50°**

White Board Activity:
Practice: Find m<MJK.

m<NKM = 180 – (88 + 48) = 44°
m<KMJ = 44°
m<MJK = 180 – (104 + 44) = 32°



Practice: Find the measure of each angle of ΔABC.

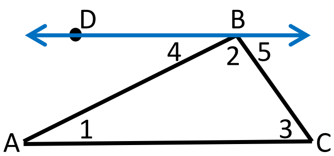


3x + 2x + 17 + 33 = 180
5x + 50 = 180
5x = 130
x = 26
m<A = 2(26) + 17 = 69°
m<B = 3(26) = 78°

An **auxiliary line** is a line that is added to a figure to aid in a proof.

The Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.



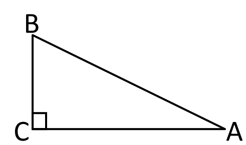
Given: Triangle ABC w/ $\overleftrightarrow{DB} \,||\, \overline{AC}$
Prove: m<1 + m<2 + m<3 = 180°

Proof:

Statements	Reasons
1. draw ℓ ||AC through B	**1. Parallel Postulate**
2. m<4 + m<2 + m<5 = 180	**2. Linear Pair Postulate (Triple)**
3. m<4 = m<1	**3. Alternate Interior Angles**
4. m<5 = m<3	**4. Alternate Interior Angles**
5. m<1 + m<2 + m<3 = 180	**5. Substitution Property**

Corollary

The acute angles of a right triangle are complementary.



Given: ΔABC is a right triangle
Prove: m<A + m<B = 90˚
Proof:

Statements	Reasons
1. m<A + m<B + m<C = 180	**1. Triangle Sum Theorem**
2. <C is a right angle	**2. Defn. of a rt. Δ**
3. m<C = 90	**3. Defn. of a rt. <**
4. m<A + m<B = 90	**4. Subt. Prop.**

Read example 2 on page 233.

Example: The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

Hint: This is the same as asking what the complement of the given angle is.
Recall: complement = 90 - angle

a. 36.7°
90 – 36.7
53.3°

b. a°
(90 – a)°

c. (2x – 4)°
90 – (2x – 4)
90 – 2x + 4
(94 – 2x)°

White Board Activity:

Practice: The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

a. 63.7° — **90 – 63.7 = 26.3°**

b. x° — **(90 – x)°**

c. 48 2/5° — **90 – 48 2/5 = 41 3/5°**

Additional Practice:

The measure of one acute angle of a right triangle is four times the measure of the other acute angle. Find the measure of each acute angle.

Let x and 4x represent the two angles

x + 4x = 90 5x = 90 x = 18° 4x = 72°

Corollary

The measure of each angle of an equiangular triangle is 60°

Explanation: If a triangle is equilateral then it is equiangular.

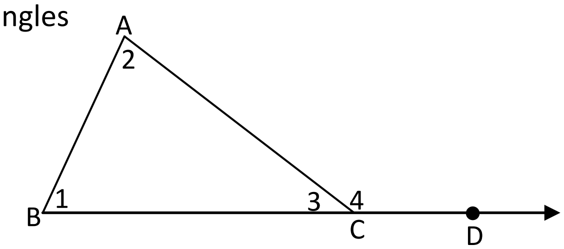
Let the measure of each angle be x, then x + x + x = 180° or 3x = 180°.

This results in x = 60°

When the sides of a triangle are extended, the three original angles are the **interior angles**. (<1, <2, <3)

The angles that are adjacent to the interior angles are the **exterior angles**. (<4)

<1 and <2 are referred to as the **remote interior angles** of exterior <4.



m<1 + m<2 + m<3 = 180 and m<3 + m<4 = 180 therefore m<1 + m<2 = m<4.

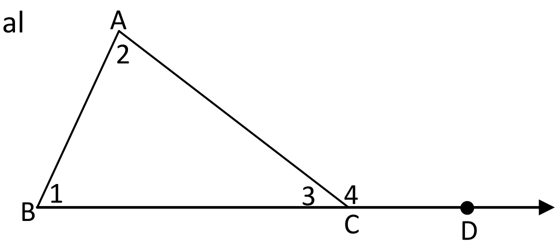
This relationship is called the **Exterior Angle Theorem**.

Exterior Angle Theorem

The measure of the exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent (remote interior) angles of the triangle.

Given: triangle ABC

Prove: m<1 + m<2 = m<4



Open the book to page 233 and read example 3.

Example: a. Given m<A = 67° and m<B = 64° find m<ACD.

m<A + m<B = m<ACD

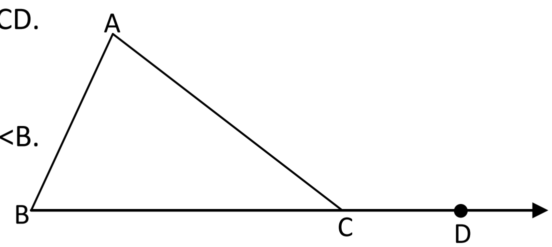
67° + 64° = 131°

b. Given m<A = 52° and m<ACD = 11° find m<B.

m<A + m<B = m<ACD

52° + m<B = 115°

m<B = 63°



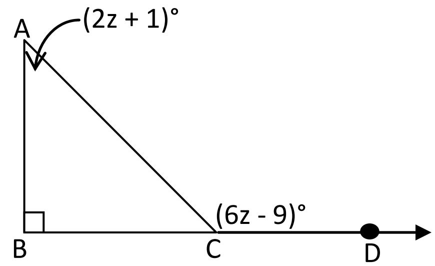
c. Given m<A = x°, m<B = 72° and m<ACD = (2x – 11)°, find the value of x. Then find the measure of the exterior angle.

72 + x = 2x – 11 83 = x

exterior angle = 2(83) – 11 = 155°

White Board Activity:

Practice: Find m<ACD.



$2z + 1 + 90 = 6z - 9$

$2z + 91 = 6z - 9$

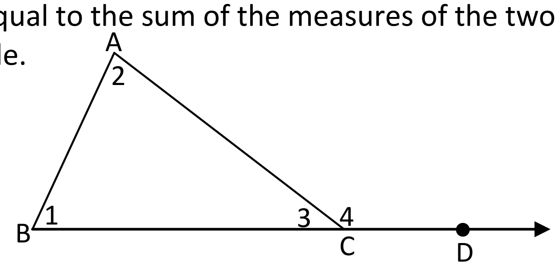
$100 = 4z$

$z = 25$

m<ACD = 6(25) – 9 = 141°

Exterior Angle Theorem

The measure of the exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent (remote interior) angles of the triangle.



Given: triangle ABC

Prove: m<1 + m<2 = m<4

Proof:

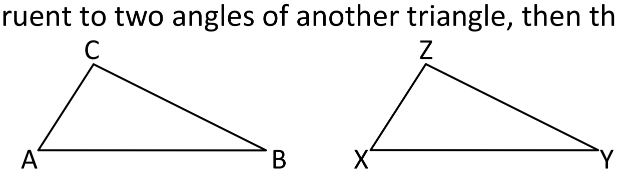
Statements	Reasons
1. ΔABC w/exterior <4	1. Given
2. m<1 + m<2 + m<3 = 180	2. Triangle Sum Theorem
3. <3 and <4 are a linear pair	3. Definition of a Linear Pair
4. m<3 + m<4 = 180	4. Linear Pair Postulate
5. m<1 + m<2 + m<3 = m<3 + m<4	5. Substitution Property
6. m<1 + m<2 = m<4	6. Subtraction Property

Third Angle Theorem

If two angles on one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.

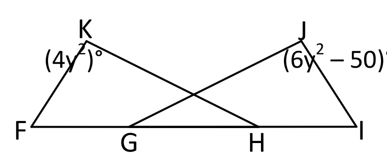
Given: <A ≅ <X, <B ≅ <Y

Prove: <C ≅ <Z



Open the book to page 234 and read example 4.

Example: Find m<K and m<J.



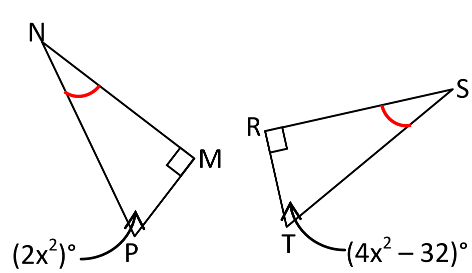
$4y^2 = 6y^2 - 40$ **$2y^2 = 50$**

$y^2 = 25$ **$y = 5$**

m<K = $4(5^2)$ = 100° and m<J = 100°

White Board Activity:

Practice: Find m<P and m<T.



$2x^2 = 4x^2 - 32$ **$2x^2 = 32$**

$x^2 = 16$ **$x = \pm 4$**

m<P = m<T = 2(16) = 32°

Assessment:

Question student pairs.

Independent Practice:

Text: pgs. 235 – 238 prob. 4 – 22, 29 – 32, 34 – 36.
Explorations: pg. 145 prob. 3 – 12.

For a Grade:

Text: pgs. 235 – 237 prob. 16, 20, 22, 34.