2.3 Paragraph Proof, Two-Column Proof, Construction proof, and Flow Chart Proof

Bell Work:
1. If $\angle 5 + \angle 8 = \angle 1 + \angle 8$, what information can be concluded? $\angle 5 = \angle 1$

2. If $\overline{mCD} + \overline{mRP} = \overline{mFG} + \overline{mRP}$, what information can be concluded? $\overline{mCD} = \overline{mFG}$

Addition Property of Equality
If $a$, $b$, and $c$ are real numbers and $a = b$, then $a + c = b + c$.

Applied to segments:
Diagram:
\[
\begin{array}{cccc}
A & B & C & D \\
\end{array}
\]
Statement: If $\overline{AB} = \overline{CD}$, then $\overline{AB} + \overline{BC} = \overline{BC} + \overline{CD}$
If $\overline{mAB} = \overline{mCD}$, then $\overline{mAB} + \overline{mBC} = \overline{mBC} + \overline{mCD}$

Applied to angles:
Diagram:
\[
\begin{array}{cccc}
A & B & C & D \\
\end{array}
\]
Statement: If $\angle AMB = \angle CMD$, then
$\angle AMB + \angle BMC = \angle BMC + \angle CMD$

Subtraction Property of Equality
If $a$, $b$, and $c$ are real numbers and $a = b$, then $a - c = b - c$.

Applied to segments:
Diagram:
\[
\begin{array}{cccc}
A & B & C & D \\
\end{array}
\]
Statement: If $\overline{AC} = \overline{BD}$, then $\overline{AC} - \overline{BC} = \overline{BD} - \overline{BC}$.
If $\overline{mAC} = \overline{mBD}$, then $\overline{mAC} - \overline{mBC} = \overline{mBD} - \overline{mBC}$

Applied to angles:
Diagram:
\[
\begin{array}{cccc}
A & B & C & D \\
\end{array}
\]
Statement: If $\angle AMC = \angle BMD$, then
$\angle AMC - \angle BMC = \angle BMD - \angle BMC$
**Reflexive Property**

If \( a \) is a real number, then \( a = a \).

Applied to segments:

Diagram:

\[
\begin{array}{c}
\text{A} \\
\text{B}
\end{array}
\]

Statement: \( AB = AB, m\overline{AB} = m\overline{AB} \) or \( \overline{AB} \cong \overline{AB} \)

Applied to angles:

Diagram:

Statement: \( m\angle A = m\angle A \) or \( \angle A \cong \angle A \)

**Substitution Property**

If \( a \) and \( b \) are real numbers and \( a = b \), then \( a \) can be substituted for \( b \).

Applied to segments:

Statement: If \( AB = 3 \text{ in} \) and \( CD = 3 \text{ in} \), then \( AB = CD \).

If \( m\overline{AB} = 3 \text{ in} \) and \( m\overline{CD} = 3 \text{ in} \), then \( m\overline{AB} = m\overline{CD} \).

Applied to angles

Statement: If \( m\angle ABC = 35^\circ \) and \( m\angle XYZ = 35^\circ \), then \( m\angle ABC = m\angle XYZ \).

**Transitive Property**

If \( a, b, \) and \( c \) are real numbers, \( a = b \) and \( b = c \), then \( a = c \).

Applied to segments:

Statement: If \( AB = CD \) and \( CD = EF \), then \( AB = EF \).

If \( m\overline{AB} = m\overline{CD} \) and \( m\overline{CD} = m\overline{EF} \), then \( m\overline{AB} = m\overline{EF} \).

Applied to angles:

Statement: If \( m\angle ABC = m\angle LMN \) and \( m\angle LMN = m\angle XYZ \), then \( m\angle ABC = m\angle XYZ \).

Remember that the Given information comes from the “if” and the Prove information comes from the “then”. This means that to make a conclusion you must be given information that matches the “if” and your conclusion must match the “then”. 
A proof is a logical series of statements and corresponding reasons that starts with the hypothesis and arrives at a conclusion.

A flow chart proof is a proof in which the steps and reasons for each step are written in boxes. Arrows connect the boxes and indicate how each step and reason is generated from one or more other steps and reasons.

Example: Given: $\overline{AB} \cong \overline{CD}$  
Prove: $\overline{AC} \cong \overline{BD}$  

- $\overline{AB} \cong \overline{CD}$  
  Given
- $m\overline{AB} = m\overline{CD}$  
  Definition of congruent segments
- $m\overline{AB} + m\overline{BC} = m\overline{AC}$  
  Segment Addition Postulate
- $m\overline{BC} = m\overline{BC}$  
  Reflexive Property
- $m\overline{AC} = m\overline{BD}$  
  Substitution Property
- $m\overline{BC} + m\overline{CD} = m\overline{BD}$  
  Segment Addition Postulate
- $m\overline{AB} + m\overline{BC} = m\overline{BC} + m\overline{CD}$  
  Addition Property of Equality
- $m\overline{AC} = m\overline{BD}$  
  Substitution Property
- $\overline{AC} \cong \overline{BD}$  
  Definition of congruent segments
A two-column proof is a proof in which the steps are written in the left column and the corresponding reasons are written in the right column. Each step and corresponding reason are numbered.

Given: \( AB \cong CD \)  
Prove: \( AC \cong BD \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AB \cong CD )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB = CD )</td>
<td>2. Definition of congruent segments</td>
</tr>
<tr>
<td>3. ( BC = BC )</td>
<td>3. Reflexive Property</td>
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<tr>
<td>4. ( AB + BC = BC + CD )</td>
<td>4. Addition Property of Equality</td>
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<tr>
<td>5. ( AB + BC = AC )</td>
<td>5. Segment Addition Property</td>
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<tr>
<td>6. ( BC + CD = BD )</td>
<td>6. Segment Addition Property</td>
</tr>
<tr>
<td>7. ( AC = BD )</td>
<td>7. Substitution Property</td>
</tr>
<tr>
<td>8. ( AC \cong BD )</td>
<td>8. Definition of congruent segments</td>
</tr>
</tbody>
</table>

A paragraph proof is a proof in which the steps and corresponding reasons are written in complete sentences.

If is given that \( AB \cong CD \), therefore by the definition of congruent segments \( AB = CD \). Now \( BC = BC \) by the reflexive property, so \( BC \) can be added to both sides of \( AB = CD \). By the addition property of equality \( AB + BC = BC + CD \). According to the segment addition property, \( AB + BC = AC \) and \( BC + CD = BD \). Now applying the substitution property to \( AB + BC = BC + CD \) results in \( AC = BD \). Again by the definition of congruent segments \( AC \cong BD \).

A construction proof is a proof that results from creating an object with specific properties using only a compass and straightedge.

Copy \( AB \) to a working line. Copy \( BC \) onto \( AB \). Copy \( CD \) to a different working line. Copy \( BC \) onto \( CD \). Copy \( AC \) to a working line. Copy \( BD \) to a working line. Use the compass to show that \( AB + BC = AC \) which in turn \( = BD \) which in turn \( = BC + CD \).
**Right Angle Congruence Theorem**

All right angles are congruent.

(If two angles are right angles, then they are congruent.)

Given: \(<ABC\) and \(<XYZ\) are right angles.

Prove: \(<ABC \cong <XYZ\)

\begin{align*}
<ABC \cong <XYZ \\
\text{Definition of congruent angles}
\end{align*}

\begin{align*}
m<ABC &= m<XYZ \\
\text{Substitution Property}
\end{align*}

\begin{align*}
m<ABC &= 90^\circ \\
\text{Definition of a right angle}
\end{align*}

\begin{align*}
m<XYZ &= 90^\circ \\
\text{Definition of a right angle}
\end{align*}

\begin{align*}
<ABC \cong <XYZ \\
\text{Definition of congruent angles}
\end{align*}
**Congruent Supplements Theorem**

If two angles are supplements of the same angle or congruent angles, then the angles are congruent.

Given: \(<ABC\) and \(<CBD\) are supplementary  
\(<WXY\) and \(<YXZ\) are supplementary  
\(<ABC \cong <WXY\)

Prove: \(<CBD \cong <YXZ\)

\(<ABC \cong <WXY\)  
\(m<ABC = m<WXY\)  
Definition of congruent segments

\(<ABC\) and \(<CBD\) are supplementary  
Given

\(<ABC\) and \(<CBD\) are supplementary  
Given

\(m<ABC + m<CBD = 180^\circ\)  
Definition of supplementary angles

\(<WXY\) and \(<YXZ\) are supplementary  
Given

\(m<WXY + m<YXZ = 180^\circ\)  
Definition of supplementary angles

\(m<ABC + m<CBD = mWX + m<YXZ\)  
Substitution Property

\(<CBD \cong <YXZ\)  
Definition of congruent segments

\(m<ABC = m<WXY\)  
Definition of congruent segments

\(<CBD = m<WXY\)  
Subtraction Property of Equality

\(<CBD \cong <YXZ\)  
Definition of congruent segments

\(<ABC \cong <WXY\)  
Given

\(m<ABC = m<WXY\)  
Definition of congruent segments
**Congruent Complements Theorem**

If two angles are complements of the same angle or congruent angles, then the angles are congruent.

Given: \( \angle ABC \) and \( \angle WXY \) are complementary  
\( \angle CBD \) and \( \angle YXZ \) are complementary  
\( \angle ABC \cong \angle WXY \)  
Prove: \( \angle CBD \cong \angle YXZ \)

**This was assigned as homework.**
Vertical Angle Theorem

If two angles are vertical angles, then they are congruent.

Given: $<1$ and $<2$ are a linear pair
$<2$ and $<3$ are a linear pair
Prove: $<1 \cong <3$

$$m<1 = m<3 \quad \text{Subtraction Property of Equality}$$

$<1$ and $<2$ are supplementary
Linear Pair Postulate

$<2$ and $<3$ are supplementary
Linear Pair Postulate

$$m<1 + m<2 = m<2 + m<3 \quad \text{Substitution Property}$$

$m<2 = m<2 \quad \text{Reflexive Property}$

$m<1 + m<2 = 180^\circ \quad \text{Definition of supplementary angles}$

$m<2 + m<3 = 180^\circ \quad \text{Definition of supplementary angles}$

$m<1 + m<2 = m<2 + m<3 \quad \text{Substitution Property}$

$$m<1 = m<3 \quad \text{Subtraction Property of Equality}$$

$m<2 = m<2 \quad \text{Reflexive Property}$

$<1 \cong <3 \quad \text{Definition of congruent angles}$