Want to play the chaos game? Dr. Michael Barnsley coined the phrase “chaos game,” in which a pattern can be created by plotting a random point within a triangle and then rolling a number cube. A few hundred rolls of the number cube result in a pattern that forms what is called the Sierpinski triangle.

Care to see if you can create a Sierpinski triangle by merely rolling a number cube? Follow these steps to see if you can do it.

1. First plot three points that will represent the vertices of a triangle. Label these points A, B, and C.
2. Plot a point anywhere inside the triangle.
3. Next, roll the number cube. If you roll a 1 or 2, measure half the distance from your initial point to vertex A and plot this point. If you roll a 3 or 4, measure half the distance from your initial point to vertex B and plot the point. If you roll a 5 or 6, measure half the distance from your initial point to vertex C and plot the point.
4. Repeat the process in Step 3, but this time, start with your new plotted point. Do this a few hundred times, and you may begin to create a Sierpinski triangle. Good luck!
Problem 1
The terms sequence and term of a sequence are defined. Ten different patterns are shown or described. Students will analyze each pattern, draw or describe additional terms in the pattern, and write a numeric sequence representing each pattern.

Grouping
- Ask a student to read the definitions. Discuss as a class.
- Have students complete each of the ten scenarios with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Positive Thinking
- How do the number of dots in each figure compare to the number of dots in the figure that follows it?
- How many dots do you think are in figure 5? Why?
- What is the total number of dots in figure 5?
- What is the total number of dots in figure 6?
- What is the total number of dots in figure 7?

Guiding Questions for Share Phase, Family Tree
- How do the number of parents in one generation compare to the number of parents in the generation that follows it?
- How many parents do you think are in the 5th generation? Why?
- What is the total number of parents in the 1st generation?
- What is the total number of parents in the 2nd generation?
- What is the total number of parents in the 3rd generation?
- What is the total number of parents in the 4th generation?
- What is the total number of parents in the 5th generation?

Problem 1 Do You See a Pattern?

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A term of a sequence is an individual number, figure, or letter in the sequence.

Examples of sequences are shown. Describe the pattern, draw or describe the next terms, and represent each sequence numerically.

“Positive Thinking”

- Analyze the number of dots. Describe the pattern. Each figure has 4 fewer dots than the figure before it.
- Draw the next three figures of the pattern.
- Write the sequence numerically to represent the number of dots in each of the first 7 figures.

25, 21, 17, 13, 9, 5, 1

Family Tree
Jessica is investigating her family tree by researching each generation, or set, of parents. She learns all she can about the first four generations, which include her two parents, her parents’ parents, her parents’ parents’ parents, and her parents’ parents’ parents’ parents.

- Think about the number of parents. Describe the pattern. Each generation has 2 times the number of parents as the generation after it.
- Determine the number of parents in the fifth and sixth generations.
  - The fifth generation has \(2^5 = 32\) parents, and the sixth generation has \(2^6 = 64\) parents.
- Write a numeric sequence to represent the number of parents in each of the 6 generations.

2, 4, 8, 16, 32, 64
Guiding Questions for Share Phase, A Collection of Squares

- How do the number of squares in figure 1 compare to the number of squares in figure 2?
- How do the number of squares in figure 2 compare to the number of squares in figure 3?
- How do the number of squares in figure 3 compare to the number of squares in figure 4?
- How many squares do you think are in figure 5? Why?
- What is the total number of squares in figure 5?
- What is the total number of squares in figure 6?
- What is the total number of squares in figure 7?

Guiding Questions for Share Phase, Al’s Omelets

- How does the number of eggs left after making 1 omelet compare to the number of eggs left after making 2 omelets?
- How does the number of eggs left after making 2 omelets compare to the number of eggs left after making 3 omelets?
- How does the number of eggs left after making 3 omelets compare to the number of eggs left after making 4 omelets?
- How many eggs do you think are left after making 5 omelets? Why?
- What is the total number of eggs left after making 4 omelets?
- What is the total number of eggs left after making 5 omelets?
- What is the total number of eggs left after making 6 omelets?
- What is the total number of eggs left after making 7 omelets?
4

Guiding Questions for Share Phase, Mario’s Mosaic

- How do the number of tiles in the 1st square compare to the number of tiles in the 2nd square?
- How do the number of tiles in the 2nd square compare to the number of tiles in the 3rd square?
- How do the number of tiles in the 3rd square compare to the number of tiles in the 4th square?
- How many tiles do you think are in the 5th square? Why?
- What is the total number of tiles in the 5th square?
- What is the total number of tiles in the 6th square?
- What is the total number of tiles in the 7th square?

Mario’s Mosaic

Mario is creating a square mosaic in the school courtyard as part of his next art project. He begins the mosaic with a single square tile. Then he adds to the single square tile to create a second square made up of 4 tiles. The third square he adds is made up of 9 tiles, and the fourth square he adds is made up of 16 tiles.

- Think about the number of tiles in each square. Describe the pattern.
  The numbers of tiles in each square are consecutive perfect squares: $1^2$, $2^2$, $3^2$, and $4^2$.

- Determine the number of tiles in the next two squares.
  The fifth square is made up of $5^2 = 25$ tiles, and the sixth square is made up of $6^2 = 36$ tiles.

- Write the sequence numerically to represent the number of tiles in each of the first 6 squares.
  $1, 4, 9, 16, 25, 36$

Guiding Questions for Share Phase, Troop of Triangles

- How do the number of shaded triangles in figure 1 compare to the number of shaded triangles in figure 2?
- How do the number of shaded triangles in figure 2 compare to the number of shaded triangles in figure 3?
- How do the number of shaded triangles in figure 3 compare to the number of shaded triangles in figure 4?
- How many shaded triangles do you think are in figure 5? Why?
- What is the total number of shaded triangles in figure 5?
- What is the total number of shaded triangles in figure 6?
- What is the total number of shaded triangles in figure 7?

Troop of Triangles

- Analyze the number of dark triangles. Describe the pattern.
  The second figure has 2 more triangles than the first, the third figure has 3 more triangles than the second, and the fourth figure has 4 more triangles than the third.

- Draw the next two figures of the pattern.

- Write the sequence numerically to represent the number of dark triangles in each of the first 6 figures.
  $1, 3, 6, 10, 15, 21$
Guiding Questions for Share Phase, Gamer Gurus

- How do the number of points after the 1st mini-game compare to the number of points after the 2nd mini-game?
- How do the number of points after the points after the 2nd mini-game compare to the number of points after the points after the 3rd mini-game?
- How do the number of points after the points after the 3rd mini-game compare to the number of points after the points after the 4th mini-game?
- How many points do you think are after the 5th mini-game? Why?
- What is the total number of points after the 4th mini-game?
- What is the total number of points after the 5th mini-game?
- What is the total number of points after the 6th mini-game?

Guiding Questions for Share Phase, Polygon Party

- How do the number of sides in figure 1 compare to the number of sides in figure 2?
- How do the number of sides in figure 2 compare to the number of sides in figure 3?

Gamer Guru

Mica is trying to beat his high score on his favorite video game. He unlocks some special mini-games where he earns points for each one he completes. Before he begins playing the mini-games (0 mini-games completed), Mica has 500 points. After completing 1 mini-game he has a total of 550 points, after completing 2 mini-games he has 600 points, and after completing 3 mini-games he has 650 points.

- Think about the total number of points Mica gains from mini-games. Describe the pattern.
  Mica gains 50 points for each mini-game he plays.
- Determine Mica’s total points after he plays the next two mini-games.
  After playing 4 mini-games, Mica has 700 points. After playing 5 mini-games, Mica has 750 points.
- Write the sequence numerically to represent Mica’s total points after completing each of the first 5 mini-games. Include the number of points he started with.
  500, 550, 600, 650, 700, 750

Polygon Party

- Analyze the number of sides in each polygon. Describe the pattern.
  Each figure is a polygon that has one more side than the previous polygon.
- Draw the next two figures of the pattern.
- Write the sequence numerically to represent the number of sides of each of the first 6 polygons.
  3, 4, 5, 6, 7, 8

- How do the number of sides in figure 3 compare to the number of sides in figure 4?
- How many sides do you think are in figure 5? Why?
- What is the total number of sides in figure 5?
- What is the total number of sides in figure 6?
- What is the total number of sides in figure 7?
Guiding Questions for Share Phase, Pizza Contest

- How does the size of the 1st slice of pizza compare to the size of the 2nd slice of pizza?
- How does the size of the 2nd slice of pizza compare to the size of the 3rd slice of pizza?
- How does the size of the 3rd slice of pizza compare to the size of the 4th slice of pizza?
- What do you think is the size of the 5th slice of pizza? Why?
- What are the fractional pieces of the pizza after the 1st cut is made?
- What are the fractional pieces of the pizza after the 2nd cut is made?
- What are the fractional pieces of the pizza after the 3rd cut is made?
- What are the fractional pieces of the pizza after the 4th cut is made?
- What are the fractional pieces of the pizza after the 5th cut is made?
- What are the fractional pieces of the pizza after the 6th cut is made?
- What are the fractional pieces of the pizza after the 7th cut is made?

Pizza Contest

Jacob is participating in a pizza-making contest. Each contestant not only has to bake a delicious pizza, but they have to make the largest pizza they can. Jacob’s pizza has a 6-foot diameter! After the contest, he plans to cut the pizza so that he can pass the slices out to share. He begins with 1 whole pizza. Then, he cuts it in half. After that, he cuts each of those slices in half. Then he cuts each of those slices in half, and so on.

- Think about the size of each slice in relation to the whole pizza. Describe the pattern.
  - After every cut, each slice is $\frac{1}{2}$ the size of each slice in the previous cut.

- Determine the size of each slice compared to the original after the next two cuts.
  - Each slice is $\frac{1}{16}$ of the original after 4 cuts. Each slice is $\frac{1}{32}$ of the original after 5 cuts.

- Write the sequence numerically to represent the size of each slice compared to the original after each of the first 5 cuts. Include the whole pizza before any cuts.
  - $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

Coin Collecting

Miranda’s uncle collects rare coins. He recently purchased an especially rare coin for $5. He claims that the value of the coin will triple each year. So even though the coin is currently worth $5, next year it will be worth $15. In 2 years it will be worth $45, and in 3 years it will be worth $135.

- Think about how the coin value changes each year. Describe the pattern.
  - Each year the coin value is 3 times greater than its value in the previous year.

- Determine the coin value after 4 years and after 5 years.
  - After four years the coin value will be equal to $135(3) = 405$, and after five years the coin value will be equal to $405(3) = 1215$.

- Write the sequence numerically to represent the value of the coin after each of the first 5 years. Include the current value.
  - $5, 15, 45, 135, 405, 1215$

Guiding Questions for Share Phase, Coin Collecting

- How does the value of the coin after the 1st year compare to the value of the coin after the 2nd year?
- How does the value of the coin after the 2nd year compare to the value of the coin after the 3rd year?
- How does the value of the coin after the 3rd year compare to the value of the coin after the 4th year?
- What do you think the value of the coin is after the 5th year? Why?
- What is the value of the coin after the 4th year?
- What is the value of the coin after the 5th year?
- What is the value of the coin after the 6th year?
- What is the value of the coin after the 7th year?
**Problem 2**

Students will organize information concerning the numeric sequences in Problem 1 in a table. They then identify the scenario from which the sequence originated, state the terms of each numeric sequence, determine if the sequence is increasing or decreasing, and verbally describe how to generate terms in each sequence. The first row of the table identifying the first scenario in Problem 1 and the essential components of that scenario are given and should serve as a model to help students complete the table.

**Grouping**
- Ask a student to read the information and directions. Discuss as a class.
- Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

**Guiding Questions for Share Phase, Questions 1 and 2**
- How many sequences could be described as increasing?
- How many sequences could be described as decreasing?
- How many sequence patterns involve multiplication or division?
- How many sequence patterns involve addition or subtraction?

**PROBLEM 2  What Do You Notice?**

There are many different patterns that can generate a sequence of numbers. For example, you may have noticed that some of the sequences in Problem 1, Do You See a Pattern? were generated by performing the same operation using a constant number. In other sequences, you may have noticed a different pattern.

The next term in a sequence is calculated by determining the pattern of the sequence, and then using that pattern on the last known term of the sequence.

1. For each sequence in Problem 1, write the problem name and numeric sequence in the table shown. Also in the table, record whether the sequence increases or decreases, and describe the operation(s) used to create each sequence. The first one has been done for you.

<table>
<thead>
<tr>
<th>Problem Name</th>
<th>Numeric Sequence</th>
<th>Increases or Decreases</th>
<th>Sequence Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Positive Thinking&quot;</td>
<td>25, 21, 17, 13, 9, 5, 1</td>
<td>Decreases</td>
<td>Begin at 25. Subtract 4 from each term.</td>
</tr>
<tr>
<td>Family Tree</td>
<td>2, 4, 8, 16, 32, 64</td>
<td>Increases</td>
<td>Begin at 2. Multiply each term by 2.</td>
</tr>
<tr>
<td>A Collection of Squares</td>
<td>49, 36, 25, 16, 9, 4, 1</td>
<td>Decreases</td>
<td>Begin at 7: Decrease each base by 1 while retaining the square.</td>
</tr>
<tr>
<td>Al’s Omelets</td>
<td>150, 144, 138, 132, 126, 120</td>
<td>Decreases</td>
<td>Begin at 150. Subtract 6 from each term.</td>
</tr>
<tr>
<td>Mario’s Mosaic</td>
<td>1, 4, 9, 16, 25, 36</td>
<td>Increases</td>
<td>Begin at 1. Increase each base by 1 while retaining the square.</td>
</tr>
<tr>
<td>Troop of Triangles</td>
<td>1, 3, 6, 10, 15, 21</td>
<td>Increases</td>
<td>Begin at 1 and add 2, then add 3, then add 4, . . .</td>
</tr>
<tr>
<td>Gamer Guru</td>
<td>500, 550, 600, 650, 700, 750</td>
<td>Increases</td>
<td>Begin at 500. Add 50 to each term.</td>
</tr>
<tr>
<td>Polygon Party</td>
<td>3, 4, 5, 6, 7, 8,</td>
<td>Increases</td>
<td>Begin at 3. Add 1 to each term.</td>
</tr>
<tr>
<td>Pizza Contest</td>
<td>1, 2, 4, 8, 16, 32</td>
<td>Decreases</td>
<td>Begin at 1. Multiply each term by $\frac{1}{2}$ (or divide each term by 2).</td>
</tr>
<tr>
<td>Coin Collecting</td>
<td>5, 15, 45, 135, 405, 1215</td>
<td>Increases</td>
<td>Begin at 5. Multiply each term by 3.</td>
</tr>
</tbody>
</table>

- Can any of the sequences be described as an even or odd sequence? Why or why not?
- Do any of the sequences begin at zero?
- Can you think of a sequence that would begin at zero? What is an example?
- What determines which value a sequence begins?
- Is the last term of the sequence you generated the end term of the sequence? Why or why not?
Problem 3
Students discuss what determines when a sequence is considered infinite or finite. Both terms are defined and the Fibonacci sequence is used as an example of an infinite sequence.

Guiding Questions for Discuss Phase, Questions 1
- Are all numbers divisible by 4?
- Can you think of a number that is not divisible by 4?
- Why are all numbers divisible by 4?
- What is the difference between an infinite sequence and a finite sequence?
- How can you determine if a sequence is infinite or finite?
- Are all sequences either infinite or finite? Why or why not?
- What is another example of an infinite sequence?
- What is another example of a finite sequence?

Do Sequences Ever End?

1. Consider a sequence in which the first term is 64, and each term after that is calculated by dividing the previous term by 4.
   Margaret says that this sequence ends at 1 because there are no whole numbers that come after 1. Jasmine disagrees and says that the sequence continues beyond 1. Who is correct? If Margaret is correct, explain why. If Jasmine is correct, predict the next two terms of the sequence.
   Jasmine is correct. Even though the sequence begins with whole numbers, this does not mean that it must contain only whole numbers. After 1, the next two terms of the sequence are \( \frac{1}{16} \) and \( \frac{1}{4} \).

2. Which sequences are similar? Explain your reasoning.
   Answers will vary.
   There are similarities based on whether the sequences increase or decrease, whether the sequences begin at 1 or not, whether the sequences are generated by adding/subtracting/multiplying/dividing by a constant, etc... For example, A Collection of Squares and Mario's Mosaic both depend on changes in the base of exponents. Positive Thinking, A Collection of Square, Ali's Omelets, and Pizza Contest decrease, but the other sequences increase.
2. Does the pattern shown represent an infinite or finite sequence? Explain your reasoning.

Answers will vary.

The pattern represents a finite sequence. There is no way to represent the next term with a figure, because numerically, the next term is 0.

3. One of the most famous infinite sequences is the Fibonacci sequence. The first 9 terms in the Fibonacci sequence are shown:

0, 1, 1, 2, 3, 5, 8, 13, 21, . . .

Explain in your own words the pattern that determines the Fibonacci sequence. Then, predict the next five terms in the sequence.

The first two terms are 0 and 1. Each subsequent term is the sum of the previous two terms. So, the next five terms in the sequence are 34, 55, 89, 144, and 233.

4. Write your own two sequences—one that is infinite and one that is finite. Describe your sequence using figures, words, or numbers. Give the first four terms of each sequence. Explain how you know that each is a sequence.

Answers will vary.

An example of an infinite sequence is 3, 7, 11, 15, . . ., which starts with 3 and increases by 4 for each subsequent term. An example of a finite sequence is a group of students, which starts with 54, then 18, then 6, then 2. Each number is divided by 3 from the previous number.

Be prepared to share your solutions and methods.