Thank Goodness Descartes Didn’t Drink Some Warm Milk!

Graphs of Sequences

LEARNING GOALS

In this lesson, you will:

- Graph arithmetic sequences.
- Graph geometric sequences.
- Recognize graphical behavior of sequences.
- Sort sequences that are represented graphically.

You’ve worked with coordinate planes before, but you may not know how they were invented. As one story goes, the 16th century French mathematician and philosopher René Descartes (pronounced day-Kart) was suffering through a bout of insomnia. While attempting to fall asleep, he spotted a fly walking on the tiled ceiling above his head. At this sight, his mind began to wander and a question popped in his head: Could he describe the fly’s path without tracing the actual path?

From that question came the revolutionary invention of the coordinate system—an invention which makes it possible to link algebra and geometry. Where have you seen examples of coordinate planes? How do coordinate planes help you identify the locations of objects?
Problem 1

Students are given an explicit formula that describes an arithmetic sequence. They will use the formula to complete a table of values listing the first ten term numbers and term values. Using the table of values, they then write each term number and term value as a set of coordinates and use the sets of coordinates to create a graph of the sequence. After describing the characteristics of the graph, the graph is used to predict a term value not shown on the graph.

Grouping
- Ask a student to read the introduction. Discuss as a class.
- Have students complete Question 1 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Question 1
- Can you use the explicit formula of the sequence to identify the type of the sequence? How?
- How did you determine the term value for the second term?
- How did you determine the term value for the third term?
- Are you using addition/subtraction or multiplication/division to generate the next term?

PROBLEM 1  Sequences as Tables and Graphs

Sometimes writers can have words flow from mind to paper without any struggles. However, that is not usually the case. For the most part, writers need to organize their thoughts, and many times they use outlines to organize these thoughts. Some of the same struggles may arise in mathematics, especially when dealing with sequences. Thus, creating a table to organize values can help you determine the sequence.

1. Consider sequence a represented by the explicit formula shown.
   \[ a_n = -10 + 4(n - 1) \]
   a. Complete the table for sequence a.

<table>
<thead>
<tr>
<th>Term Number ((n))</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−10</td>
</tr>
<tr>
<td>2</td>
<td>−6</td>
</tr>
<tr>
<td>3</td>
<td>−2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>22</td>
</tr>
<tr>
<td>10</td>
<td>26</td>
</tr>
</tbody>
</table>
   b. Write each pair of numbers from the table as an ordered pair. Let the independent variable represent the term number, and let the dependent variable represent the term value.
   \((1, −10), (2, −6), (3, −2), (4, 2), (5, 6), (6, 10), (7, 14), (8, 18), (9, 22), (10, 26)\)

- What does the operation you are using to generate the next term tell you about the type of sequence?
- Does the sequence have a common difference or a common ratio?
Guiding Questions for Share Phase, Question 1

- Does the graph of the sequence pass through the origin? How do you know?
- What quadrants are used to graph the sequence?
- Would you describe the graph of the sequence as increasing or decreasing? Why?
- If only integers are used as the term numbers, what does this tell you about the graph?

c. Graph the ordered pairs on the grid shown and label the axes.

```
<table>
<thead>
<tr>
<th>Term Number (n)</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>-4</td>
</tr>
</tbody>
</table>
```

d. Describe the shape of the graph.
The graph is a line.

e. Is the graph discrete or continuous? Explain your reasoning.
The graph is discrete because the terms are integer values beginning at 1.

f. Can you use the graph to predict the 20th term? Explain your reasoning.
Answers will vary.
Some students may be able to use their graph to predict the 20th term, but others may have to change the bounds to predict the 20th term. The 20th term is 66.
Grouping

Have students complete Question 2 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Question 2

- Can you use the explicit formula of the sequence to identify the type of the sequence? How?
- How did you determine the term value for the first term?
- How did you determine the term value for the second term?
- Are you using addition/subtraction or multiplication/division to generate the next term?
- What does the operation you are using to generate the next term tell you about the type of sequence?
- Does the sequence have a common difference or a common ratio?

2. Consider sequence $g$ represented by the explicit formula shown.

$$g_n = 1, \quad g_n = 2^{n-1}$$

a. Create a table of values using the first ten terms of sequence $g$.

<table>
<thead>
<tr>
<th>Term Number ($n$)</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
</tr>
<tr>
<td>8</td>
<td>128</td>
</tr>
<tr>
<td>9</td>
<td>256</td>
</tr>
<tr>
<td>10</td>
<td>512</td>
</tr>
</tbody>
</table>

b. Write each pair of numbers from the table as an ordered pair. Let the independent variable represent the term number, and let the dependent variable represent the term value.

$(1, 1), (2, 2), (3, 4), (4, 8), (5, 16), (6, 32), (7, 64), (8, 128), (9, 256), (10, 512)$
Guiding Questions for Share Phase, Question 2

- Does the graph of the sequence pass through the origin? How do you know?
- What quadrants are used to graph the sequence?
- Would you describe the graph of the sequence as increasing or decreasing? Why?
- If only integers are used as the term numbers, what does this tell you about the graph?
- How is this graph different than the graph in the previous question?
- How is this graph similar to the graph in the previous question?

c. Graph the ordered pairs on the grid shown and label the axes.

![Graph of ordered pairs](image)

d. Describe the shape of the graph.
   - The graph is an upward curve that appears to be exponential.

e. Is the graph discrete or continuous? Explain your reasoning.
   - The graph is discrete because the terms are integer values beginning at 1.

f. Can you use the graph to predict the 20th term? Explain your reasoning.
   - Answers will vary.
   - Very few, if any, students will be able to use their graph to predict the 20th term. Most will have to change the bounds to predict the 20th term. The 20th term is 524,288.
**Problem 2**

Students cut out 12 graphs and 12 graphic organizers at the end of this problem. Using the 12 sequences from Lesson 4.2, students will paste the appropriate sequence and graph on each of the graphic organizers. They then complete each graphic organizer by writing the explicit and recursive formulas that represent each sequence.

**Grouping**

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

**Guiding Questions for Share Phase, Questions 1 through 3**

- Which graphs appear to be linear? What information does this give you about the sequence?
- Which graphs appear to be exponential? What information does this give you about the sequence?
- Which graphs appear to be increasing? What information does this give you about the sequence?
- Which graphs appear to be decreasing? What information does this give you about the sequence?
- How can determining the $y$-intercept be helpful in matching the graphs to the appropriate sequence?
- How can the coordinates of the first term be helpful in matching the graphs to the appropriate sequence?
2. Did knowing whether the sequences were increasing or decreasing help you to match the graphs to their corresponding sequences? Explain your reasoning.
   Answers will vary.
   Students may not have used this property to match the graphs, but those who did would have probably matched their graphs in a more efficient manner.

3. What other strategies did you use to match the graphs to their corresponding sequences?
   Answers will vary.
   - Determining the bounds of the $y$-axis,
   - Determining the $y$-intercepts,
   - Determining the coordinates of the first term,
   - Determining the coordinates of later terms (7th through 10th),
   - Determining which were linear or exponential.

Be prepared to share your solutions and methods.
Sequence

\[ A \]
\[45, 90, 180, 360, 720, 1440, 2880, \ldots\]

- multiply by 2
- geometric: \( r = 2 \)

Explicit Formula

\[ g_n = g_1 \cdot r^{n-1} \]
\[ g_n = 45 \cdot 2^{n-1} \]

Sequence Type: Geometric

Graph

Recursive Formula

\[ g_n = 45 \]
\[ g_n = 2 \cdot g^{n-1} \]
Sequence

-4, -2, 0, 2, 4, 6, 8, ...
add 2
arithmetic: $d = 2$

Explicit Formula

\[ a_n = a_1 + d(n - 1) \]
\[ a_n = -4 + 2(n - 1) \]
\[ = -4 + 2n - 2 \]
\[ = 2n - 6 \]

Sequence Type: Arithmetic

Recursive Formula

\[ a_n = a_{n-1} + 2 \]
\[ a_n = 2 + a_{n-1} \]
Sequence

\[ C \]

\[ -2, -6, -18, -54, -162, -486, -1458, \ldots \]
multiply by 3
geometric: \( r = 3 \)

Explicit Formula

\[ g_n = g_1 \cdot r^{n-1} \]

\[ g_n = -2 \cdot 3^{n-1} \]

Graph

Recursive Formula

\[ g_n = a \]

\[ g_n = g_{n-1} \cdot 3 \]
Sequence

\[ a_1 = 4 \]

\[ a_n = a_{n-1} - \frac{9}{4} \]

Explicit Formula

\[ a_n = a_1 + d(n - 1) \]

\[ a_n = 4 + \left( -\frac{9}{4} \right)(n - 1) \]

\[ a_n = 4 - \frac{9}{4}n + \frac{9}{4} \]

\[ a_n = -\frac{9}{4}n + \frac{25}{4} \]

Sequence Type: Arithmetic

Graph

Recursive Formula
Sequence

\[ F \]

1234, 123.4, 12.34, 1.234, 0.1234, 0.01234, \ldots

- multiply by 0.1
- geometric; \( r = 0.1 \)

**Explicit Formula**

\[ g_n = g_1 \cdot r^{n-1} \]

\[ g_n = 1234 \cdot (0.1)^{n-1} \]

**Sequence Type:** Geometric

**Recursive Formula**

\[ g_n = g_{n-1} \cdot 0.1 \]

### Graph

- Points plotted on a graph with values ranging from -200 to 1300 on the vertical axis and from 0 to 10 on the horizontal axis.

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Sequence

\[ H \]
-20, -16, -12, -8, -4, 0, 4, 8, \ldots

add 4

arithmetic: \( d = 4 \)

Explicit Formula

\[ a_n = a_1 + d(n - 1) \]

\[ a_n = -20 + 4(n - 1) \]

\[ = -20 + 4n - 4 \]

\[ = 4n - 24 \]

Sequence Type:

Arithmetic

Graph

Recursive Formula

\[ a_1 = -20 \]

\[ a_n = 4 + a_{n-1} \]
Sequence Type: Geometric

1, 10, 100, 1000, 10,000, 100,000, . . .
multiply by 10
geometric: $r = 10$

Explicit Formula

$g_n = g_1 \cdot r^{n-1}$
$g_n = 1 \cdot 10^{n-1}$
$g_n = 10^{n-1}$

Recursive Formula

$g_n = 1$
$g_n = 10 \cdot g_{n-1}$

Graph
Sequence

\[J\]

-5, \( \frac{5}{2} \), \( -\frac{5}{4} \), \( \frac{5}{8} \), \( -\frac{5}{16} \), \( \frac{5}{32} \), \ldots

multiply by \( \frac{1}{2} \)

geometric: \( r = \frac{1}{2} \)

Explicit Formula

\[g_n = g_1 \cdot r^{n-1}\]

\[g_n = -5 \cdot \left( \frac{1}{2} \right)^{n-1}\]

Sequence Type:

Geometric

Graph

Recursive Formula

\[g_1 = -5\]

\[g_n = \frac{1}{2} \cdot g_{n-1}\]
Sequence

\[ 6.5, 5, 3.5, 2, 0.5, -1, -2.5, \ldots \]

subtract 1.5

arithmetic: \( d = -1.5 \)

Explicit Formula

\[ a_n = a_1 + d(n - 1) \]

\[ a_n = 6.5 + (-1.5)(n - 1) \]

\[ = 6.5 - 1.5n + 1.5 \]

\[ = -1.5n + 8 \]

Sequence Type:

Arithmetic

Graph

Recursive Formula

\[ a_1 = 6.5 \]

\[ a_n = -1.5 + a_{n-1} \]
Sequence

\[ M \]
-16, 4, \(-1\), \(-\frac{1}{4}\), \(-\frac{1}{16}\), \(-\frac{1}{64}\), \ldots

divide by \(-4\)

g{}^{1} \text{ geometric: } r = -\frac{1}{4}

Explicit Formula

\[ g_1 = g_0 \cdot r^{1-1} \]
\[ g_n = -16 \cdot \left(-\frac{1}{4}\right)^{n-1} \]

Sequence Type: Geometric

Graph

\[ g_1 = -16 \]
\[ g_n = -\frac{1}{4} \cdot g_{n-1} \]

Recursive Formula
Sequence

N
1473.2, 1452.7, 1432.2, 1411.7, 1391.2, 1370.7, 1350.2, ... 
subtract 20.5
arithmetic: \( d = -20.5 \)

Explicit Formula

\[ a_n = a_1 + d(n - 1) \]
\[ a_n = 1473.2 + (-20.5)(n - 1) \]
\[ = 1473.2 - 20.5n + 20.5 \]
\[ = -20.5n + 1493.7 \]

Recursive Formula

\[ a_n = 1473.2 \]
\[ a_n = -20.5 + a_{n-1} \]
Sequence

$p$

\(-4, 12, -36, 108, -324, 972, \ldots\)

multiply by \(-3\)

geometric: \(r = -3\)

Sequence Type: Geometric

Explicit Formula

\[ g_n = g_1 \cdot r^{n-1} \]

\[ g_n = -4 \cdot (-3)^{n-1} \]

Recursive Formula

\[ g_1 = -4 \]

\[ g_n = -3 \cdot g_{n-1} \]

Graph