Review Problems for Final Exam

Determine whether the two expressions are equivalent.

1. $6n + 8$ and $2(3n + 4)$

2. $(n^2 + 4n) - n^2$ and $4n$

3. $(5x + 3) + 3x^2 - 2$ and $(5x + 1) + 3x^2$

4. $(7x^2 + 1) - (3x - 1)(x + 4)$ and
$2x(x - 3) + 2x^2 - 5x + 5$

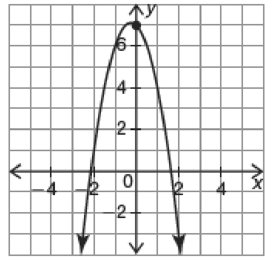
5. $(2x + 1)^2 - 2x(x - 3)$ and $6x^2 + 6x + 2 - (2x - 1)^2$

6. Convert the quadratic function in factored form to standard form.
$f(x) = (x + 2)(x + 9)$

7. Convert the quadratic function in vertex form to standard form.
$f(x) = -2(x + 1)^2 - 5$

Circle the function that matches each graph. Explain your reasoning.

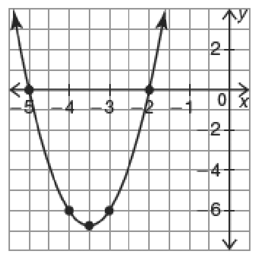
8.



$f(x) = 2x^2 - x + 7$ $\qquad$ $f(x) = -x^2 - 2x + 7$

$f(x) = -2x^2 - x + 7$ $\qquad$ $f(x) = -2x^2 - x - 2$

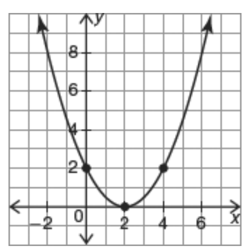
9.



$f(x) = -3(x + 2)(x - 5)$ $\qquad$ $f(x) = 3(x - 2)(x - 5)$

$f(x) = 3(x + 2)(x + 5)$ $\qquad$ $f(x) = -3(x - 2)(x - 5)$

10.



$f(x) = -\frac{1}{2}(x - 2)^2$ $\qquad$ $f(x) = \frac{1}{2}(x - 2)^2$

$f(x) = \frac{1}{2}(x - 2)^2 + 2$ $\qquad$ $f(x) = \frac{1}{2}(x + 2)^2$

Each given function is in transformational function form $g(x) = Af(B(x - C)) + D$, where $f(x) = x^2$. Identify the values of *C* and *D* for the given function. Then, describe how the vertex of the given function compares to the vertex of *f*(*x*).

11. $g(x) = f(x - 5) - 11$

12. $g(x) = f(x + 2) + 3$

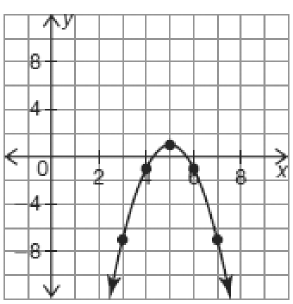
13. Graph the vertical dilation of $f(x) = x^2$ and tell whether the transformation is a vertical stretch or a vertical compression and if the graph includes a reflection.

$$g(x) = -\frac{1}{2}x^2 - 3$$

14. The given function is in transformational function form $g(x) = Af(B(x - C)) + D$, where $f(x) = x^2$. Describe how *g*(*x*) compares to *f*(*x*). Then, use coordinate notation to represent how the *A*-, *C*-, and *D*-values transform *f*(*x*) to generate *g*(*x*).

$$g(x) = \frac{1}{3}f(x - 6) - 3$$

15. Write the function that represents the graph.



16. Simplify: $9 + 3i(7 - 2i)$

For each complex number, write its conjugate.

17. $3 + 5i$

18. $-7i$

19. Determine the product. $(4i - 5)(4i + 5)$

Use the discriminant to determine whether each function has real or imaginary zeros.

20. $f(x) = -3x + x - 9$

21. $f(x) = 9x^2 - 12x + 4$

Use the vertex form of a quadratic equation to determine whether the zeros of each function are real or imaginary. Explain how you know.

22. $f(x) = -2(x - 1)^2 - 5$

23. $f(x) = \frac{3}{4}(x + 4)^2 - 6$

Factor each function over the set of real or imaginary numbers. Then, identify the type of zeros.

24. $n(x) = x^2 - 5x - 14$

25. $g(x) = x^2 + 6x + 10$

Determine the product of linear and quadratic factors. Verify graphically that the expressions are equivalent.

26. $(2x - 9)(4x^2 - 5x - 12)$

27. $7x(x + 5)^2$

28. $(x^2 + 1)(8 - x)$

Sketch the graph of $f(x)$ and describe the end behavior of each graph.

29. x^3

30. $-x^2$

The equation for a polynomial function *p*(*x*) is given. The equation for the transformed function *m*(*x*) in terms of *p*(*x*) is also given. Describe the transformation(s) performed on *p*(*x*) that produced *m*(*x*). Then, write a specific equation for *m*(*x*).

31. $p(x) = x^3$; $m(x) = 4p(x - 3) - 5$

32. $p(x) = x^5$; $m(x) = 0.5p(-x) + 4$

Use a coordinate plane to sketch a graph with the given characteristics. If the graph is not possible to sketch, explain why.

33. Characteristics:

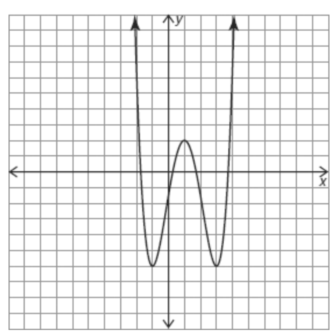
- even degree
- increases to $x = -2$, then decreases to $x = 0$, then increases to $x = 2$, then decreases
- relative minimum at $y = 1$
- two absolute maximums at $y = 4$

34. Characteristics:

- as $x \to \infty, f(x) \to \infty$

  as $x \to -\infty, f(x) \to -\infty$
- *y*-intercept at $y = -2$
- three *x*-intercepts
- two relative extrema

35. Circle the function(s) that could model each graph. Describe your reasoning for either eliminating or choosing each function.



$f(x) = x^4 - 4x^3 - 2x^2 + 12x - 3$

$f(x) = 2(x + 3)(x + 4)$

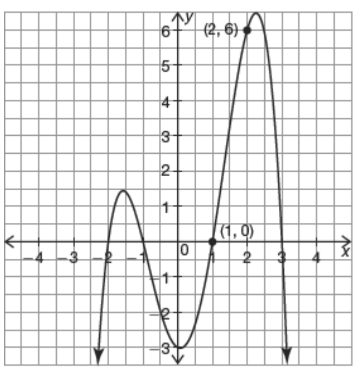
$f(x) = -2x^5 + x^4 - 3x^3 + 12$

Sketch a set of functions whose product builds a cubic function with the given characteristics. Explain your reasoning.

36. The cubic function is in Quadrants I and III only

37. two imaginary zeros and a real zero $x = 0$

38. Determine the average rate of change for the interval (1, 2) for the polynomial function.



Determine each function value using the Remainder Theorem. Explain your reasoning.

39. Determine $p(-2)$ if $p(x) = x^4 - 10x^3 + 8x^2 + 106x - 105$.

40. Determine $p(-3)$ if $p(x) = 2x^4 + 5x^3 + 8x^2 + 15x + 6$.

41. Determine $p(1)$ if $p(x) = x^4 + 3x^3 - 6x^2 - 8x$.

Use the Factor Theorem to determine whether the given expression is a factor of each polynomial. Explain your reasoning.

42. Is $x - 3$ a factor of $f(x) = 4x^4 - x^3 - 52x^2 - 35x + 12$?

43. Is $3x + 4$ a factor of $f(x) = 3x^3 + 13x^2 + 18x + 8$?

44. Use the Factor Theorem to determine $g(x) = (x - 2)(x + 9)(x + 1)$ is the factored form of $f(x) = x^3 + 8x^2 - 11x - 18$

Factor completely.

45. $x^2 - 16x - 36$

46. $x^3 + x^2 - 4x - 4$

47. $x^4 - 3x^3 - x^2 - 3x$

48. $x^4 - 29x^2 + 100$

49. $x^3 - 8y^3$

50. $x^3 + 64y^3$

51. $x^4 - 36$

52. $49x^2 - 4y^2$

53. $9x^4 + 42x^2y + 49y^2$

54. $8x^4 - 16x^3 + 56x^2 - 24x$

55. $25x^2 - 35x + 12$

Use the Rational Root Theorem to determine the possible rational roots for each polynomial equation. Then, solve completely. Use the graph, if given, to identify possible zeros.

56. $x^3 + 3x^2 - 18x - 40$

57. $x^2 - 8x + 17$

Perform each calculation. Describe any restriction(s) for the value of *x* and simplify the answer when possible.

58. $\frac{3}{x} + \frac{1}{x+1}$

59. $\frac{1}{x+3} - \frac{1}{x-3}$

60. $\frac{x+1}{x^2-16} - \frac{x}{x^2+7x+12}$

61. $\frac{1}{2x^2+3x-2} \cdot \frac{x^2-2x-8}{x-4}$

62. $\frac{x^2+6x+8}{3x+2} \div \frac{-x-4}{3x^2-x-2}$

Solve each rational equation. Describe any restrictions for the value of *x*. Check your answer(s) and identify any extraneous roots should they occur.

63. $\frac{9}{x-3} = \frac{27}{x^2-3x} + \frac{6}{x}$

64. $\frac{x-3}{x^2} = \frac{x-3}{x^2-1}$

65. $\frac{2}{x} - \frac{3}{2x} = \frac{1}{x^2}$

66. $\frac{x}{x+2} + \frac{4x+6}{2x^2+5x+3} = \frac{x-1}{2x+4}$

Write an equation to model each scenario. Then, solve each equation.

67. The directions on the back of a 2 quart bottle of a 60% orange concentrate says it needs to be mixed with water to obtain a 20% orange drink. How much water should Hector add to the concentrate to obtain a drink that is 20% orange concentrate?

68. Kendall can wash 24 golf carts in a 4 hour shift. If Benny helps him they can get the job done in 2 hours. How long will it take Benny to do the job by himself?

69. Nyesha owns a lawn service company. Currently it takes her 50 hours a week to service all of her customers. To reduce the number of hours a week she needs to work, Nyesha hires Shantese to help her. While Nyesha was on vacation, Shantese was able to complete all of the work in 60 hours. If Shantese and Nyesha work together, after Nyesha returns from vacation, how long will it take them to service all their customers?

Determine the inverse of each function.

70. $f(x) = 4x^2 - 1$

71. $f(x) = 3x^3 + 2$

72. $f(x) = (x+2)^4 - 16$

73. Determine the inverse and describe the domain and range of the inverse. $f(x) = 2\sqrt[5]{x} - 1$

Perform the indicated operation. Be sure to simplify.

74. $\sqrt{16x^{10}y^8z^2}$

75. $5\sqrt{x}(\sqrt{x} + 3\sqrt{x}) - 10x$

76. $(5\sqrt{x})^2(4\sqrt{4x})$

77. $\frac{5\sqrt{2x^2y^2}}{3\sqrt[4]{32x^2y^5}}$

78. $5\sqrt{x} + 4\sqrt{x} + \sqrt[4]{x}$

Solve each radical equation. Check for extraneous solutions.

79. $2x - 2 = \sqrt{x+2}$

80. $\sqrt[5]{3x-3} = 2$

81. $\sqrt{9x+3} - 11 = -8$

Simplify each expression completely, using only positive exponents.

82. $\dfrac{(2xy^4)^3}{(x^2y^3)^4}$

83. $(2x^{-2}y^3)^{-2}$

84. $\dfrac{x^3 \cdot y^{-5}}{x^{-4} \cdot y^2}$

Evaluate each log.

85. $\log_3 81$

86. $\log_4 4^5$

87. Write a sentence to demonstrate how the equation $x = \log_4 6$ would be spoken. Then convert the logarithmic equation to exponential form.

Write each logarithmic expression in expanded form.

88. $\log_5 10x^9y^7$

89. $\log_3 \dfrac{2}{x^2y^{10}}$

Write each logarithmic expression using a single logarithm. Evaluate the logarithm if possible.

90. $\log_3 6 + 5\log_3 x$

91. $\log_7 122 + 5\log_7 x + 8\log_7 y$

Convert each number of radians to degrees.

92. $\dfrac{7\pi}{4}$

93. $\dfrac{11\pi}{6}$

Convert each number of degrees to radians.

94. $240°$

95. $135°$

Evaluate each trigonometric function.

96. $\sin\frac{5\pi}{4}$

97. $\cos\frac{2\pi}{3}$

Calculate the tangent of each angle given the cosine and sine of the angle.

98. $\sin\theta=\frac{8}{17}$, $\cos\theta=\frac{15}{17}$

99. $\sin\theta=\frac{2\sqrt{5}}{5}$, $\cos\theta=\frac{\sqrt{5}}{5}$

100. Evaluate the tangent function $\tan\frac{5\pi}{4}$ by using the relationship between the tangent function and the sine and cosine functions.

Determine 2 coterminal angles for the following angles

101. $\theta = 30°$

102. $\theta = 45°$

103. $\theta = 240°$

Determine each of the following in the first revolution. Determine *x* in radians.

104. $\cos x = -\frac{\sqrt{3}}{2}$

105. $\tan x = 0$

106. Find the amplitude of $y = 3\sin x$

107. Find the amplitude of $y = \frac{1}{2}\cos x$

108. Write an arithmetic sequence that satisfies the criteria: The first term is 0.64. The common difference is -0.25. Write the first six terms of the sequence.

109. Write an arithmetic sequence for the following situation: Rosa is training for a bicycle race. She rides 4 miles the first day. Every day after that she increses the distance by half of a mile. What are the distances she rides for the first four days?

Determine each quotient using polynomial long division. Write the dividend as the product of the divisor and the quotient plus the remainder.

110. $2x-1\overline{\smash{)}\,2x^3-7x^2-19x+11}$

111. $x-2\overline{)2x^3-x^2-13x-6}$

Determine the possible rational roots of each polynomial using the Rational Root Theorem.

112. $2x^4-4x^2+15$

113. $-2x^3+5x+18$

Determine whether each geometric series is convergent or divergent and calculate the series.

114. $\sum_{i=1}^{\infty}\left(\frac{2}{7}\right)^i$

115. $\sum_{i=0}^{\infty}100(0.1)^i$

Write an explicit formula for the *n*th term of the geometric sequence that models each problem situation. Identify and interpret the meaning of the first term, g_1, and the common ratio, *r*.

116. Panna initially invested \$1500 in her 401K plan. Due to the slow economy, her investment began to decrease by 1.5% per year.

Use your knowledge of geometric sequences and series to solve each problem.

117. Contagious diseases like the flu can spread rather rapidly. Suppose in a large community initially 3 people have the flu. By the end of the 1st week, 12 people have the flu. If this pattern continues 786,432 will have the flu by the end of the 10th week. What is the total number of people who will have the flu during the 10-week period?

Solve each logarithmic equation. Check your answer(s).

118. $\log_{15}(x^2-2x)=1$

Use the properties of logarithms to solve each logarithmic equation. Check your answer(s).

119. $2\log_3 x-\log_3 8=\log_3(x-2)$

120. $\log_4(x+3)+\log_4 x=1$

121. Sketch the graph of a normal distribution.

122. Describe the type of sampling performed in each scenario.

a. Jeremy wants to perform an experiment to analyze the weights of players on his soccer team. He lists the weights of the players from largest to smallest and takes a sample of the first ten players on the list.

b. Minnie wants to perform an experiment to analyze the test scores of students in her math class. She asks ten students to volunteer their scores and uses those as a sample for her experiment.

c. Devin wants to perform an experiment to analyze the heights of apple trees in an orchard. He assigns each apple tree in the orchard a unique number and then uses a random number generator to choose a sample of ten trees.

d. Rachel wants to perform an experiment to analyze the expiration dates of cartons of milk on a shelf in a grocery store. She takes ten containers of milk from the front of the shelf and records their dates.

Review Problems for Final Exam
Answer Section

1. The expressions are equivalent.

Use the distributive property and combine like terms.

$2(3n + 4) = 6n + 8.$

2. The expressions are equivalent.

Combine like terms. $(n^2 + 4n) - n^2 = 4n.$

3. $(5x + 3) + 3x^2 - 2$ | $(5x + 1) + 3x^2$

$3x^2 + 5x + 1$ | $3x^2 + 5x + 1$

The expressions are equivalent.

4. $(7x^2 + 1) - (3x - 1)(x + 4)$

$7x^2 + 1 - (3x^2 + 11x - 4)$

$7x^2 + 1 - 3x^2 - 11x + 4$

$4x^2 - 11x + 5$

$2x(x - 3) + 2x^2 - 5x + 5$

$2x^2 - 6x + 2x^2 - 5x + 5$

$4x^2 - 11x + 5$

The expressions are equivalent.

5. $(2x + 1)^2 - 2x(x - 3)$

$4x^2 + 4x + 1 - 2x^2 + 6x$

$2x^2 + 10x + 1$

$6x^2 + 6x + 2 - (2x - 1)^2$

$6x^2 + 6x + 2 - (4x^2 - 4x + 1)$

$6x^2 + 6x + 2 - 4x^2 + 4x - 1$

$2x^2 + 10x + 1$

The expressions are equivalent.

6. $f(x) = x^2 + 9x + 2x + 18$

$\quad = x^2 + 11x + 18$

7. $f(x) = -2(x^2 + 2x + 1) - 5$

$\quad = -2x^2 - 4x - 7$

8. $f(x) = -2x^2 - x + 7$

The *a*-value is negative so the parabola opens down. Also, the *y*-intercept is (0, 7).

9. $f(x) = 3(x + 2)(x + 5)$

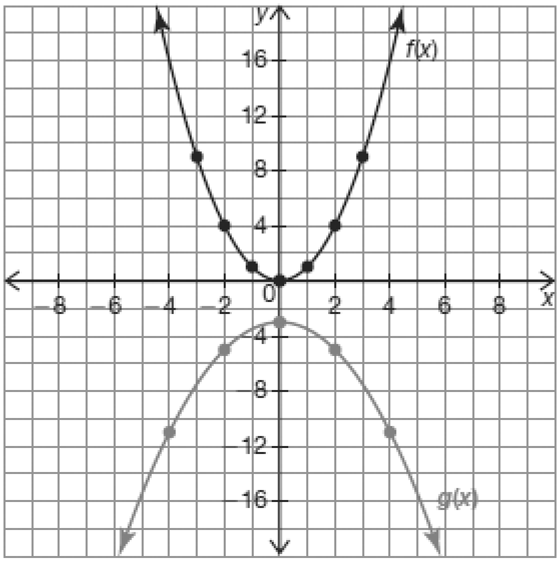
The *a*-value is positive so the parabola opens up. The *x*-intercepts are -5 and -2.

10. $f(x) = \frac{1}{2}(x - 2)^2$

The *a*-value is positive so the parabola opens up. The minimum point is (2, 0).

11. The *C*–value is 5 and the *D*–value is $-11,$ so the vertex will be shifted 5 units to the right and 11 units down to $(5, -11).$

12. The *C*–value is -2 and the *D*–value is 3 so the vertex will be shifted 2 units to the left and 3 units up to $(-2, 3).$

13. 

vertical compression and reflection over the x-axis, and shifted down 3.

14. The *A*–value is $\frac{1}{3},$ so the graph will have a vertical compression by a factor of $\frac{1}{3}.$ The *C*–value is 6 and the *D*–value is -3 so the vertex will be shifted 6 units to the right and 3 units down to $(6, -3).$

$$(x, y) \rightarrow (x + 6, \frac{1}{3}y - 3)$$

15. $f(x) = -2(x - 5)^2 + 1$

16. $$\begin{aligned} 9 + 3i(7 - 2i) &= 9 + 21i - 6i^2 \\ &= 9 + 21i - 6(-1) \\ &= (9 + 6) + 21i \\ &= 15 + 21i \end{aligned}$$

17. $3 - 5i$

18. $7i$

19. $$\begin{aligned} (4i - 5)(4i + 5) &= 16i^2 + 20i - 20i - 25 \\ &= 16(-1) - 25 \\ &= -41 \end{aligned}$$

20. $$\begin{aligned} b^2 - 4ac &= 1^2 - 4(-3)(-9) \\ &= 1 - 108 \\ &= -107 \end{aligned}$$

The discriminant is negative, so the function has imaginary zeros.

21. $b^2 - 4ac = (-12)^2 - 4(9)(4)$

$$= 144 - 144$$

$$= 0$$

The discriminant is zero, so the function has real zeros (double roots).

22. Because the vertex $(-1, -5)$ is below the *x*-axis and the parabola is concave down $(a < 0)$, it does not intersect the *x*-axis. So, the zeros are imaginary.

23. Because the vertex $(-4, -6)$ is below the *x*-axis and the parabola is concave up, it intersects the *x*-axis. So, the zeros are real.

24. $n(x) = (x - 7)(x + 2)$

$x = 7, x = -2$

The function $n(x)$ has two real zeros.

25. $g(x) = \left[x - (-3 - i)\right]\left[x - (-3 + i)\right]$

$x = -3 - i, x = -3 + i$

The function $g(x)$ has two imaginary zeros.

26. $(2x - 9)(4x^2 - 5x - 12) = 8x^3 - 10x^2 - 24x - 36x^2 + 45x + 108$

$$= 8x^3 - 46x^2 + 21x + 108$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

27. $7x(x + 5)^2 = 7x(x + 5)(x + 5)$

$$= 7x(x^2 + 5x + 5x + 25)$$

$$= 7x(x^2 + 10x + 25)$$

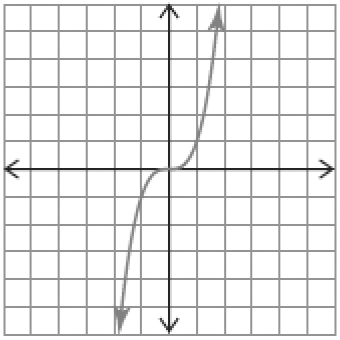
$$= 7x^3 + 70x^2 + 175x$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

28. $(x^2 + 1)(8 - x) = 8x^2 - x^3 + 8 - x$

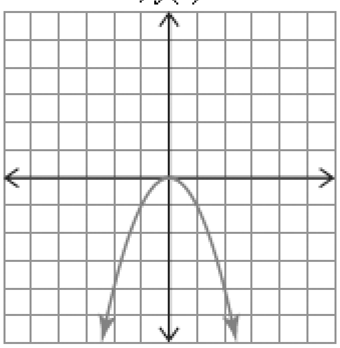
$$= -x^3 + 8x^2 - x + 8$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

29. 

As $x \to \infty,\ f(x) \to \infty$

As $x \to -\infty,\ f(x) \to -\infty$

30. 

As $x \to \infty,\ f(x) \to -\infty$

As $x \to -\infty,\ f(x) \to -\infty$

31. $(x,\ y) \to (x+3,\ 4y-5)$

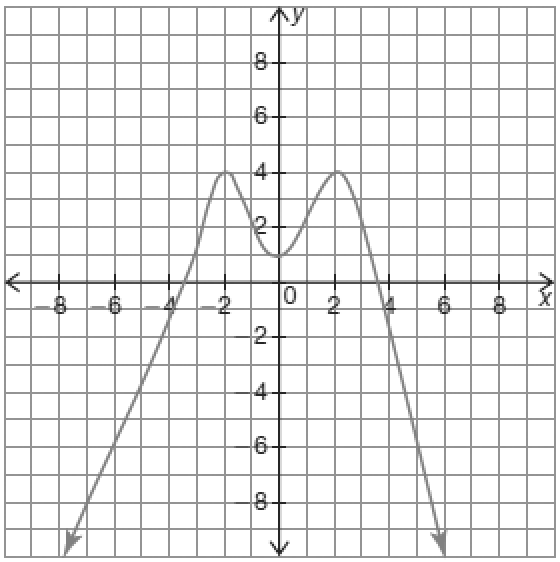
The graph of the function $m(x)$ is translated 3 units to the right and 5 units down. It has also been stretched vertically by a factor of 4.

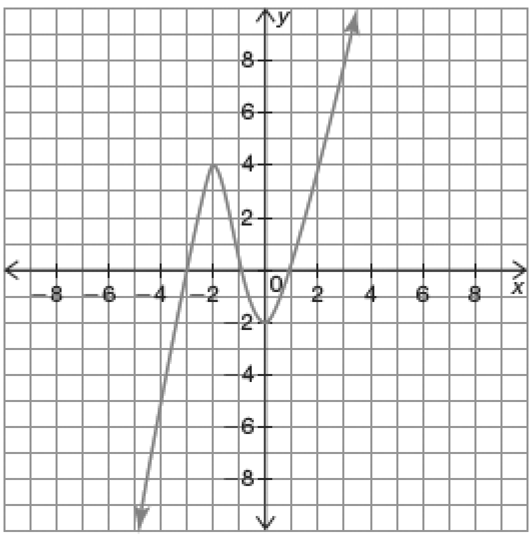
$$\begin{aligned} m(x) &= 4p(x-3)-5 \\ &= 4(x-3)^3-5 \\ &= 4(x^3-9x^2+27x-27)-5 \\ &= 4x^3-36x^2+108x-113 \end{aligned}$$

32. $(x,\ y) \to (-x,\ 0.5y+4)$

The graph of the function $m(x)$ is vertically compressed by a factor of 0.5, translated 4 units up and reflected about the y-axis.

$$\begin{aligned} m(x) &= 0.5p(-x)+4 \\ &= 0.5(-x)^5+4 \\ &= -0.5x^5+4 \end{aligned}$$

33. 

34. 

35. $f(x) = x^4 - 4x^3 - 2x^2 + 12x - 3$

I chose this function because it represents an even function greater than degree 2 with a positive *a*-value.

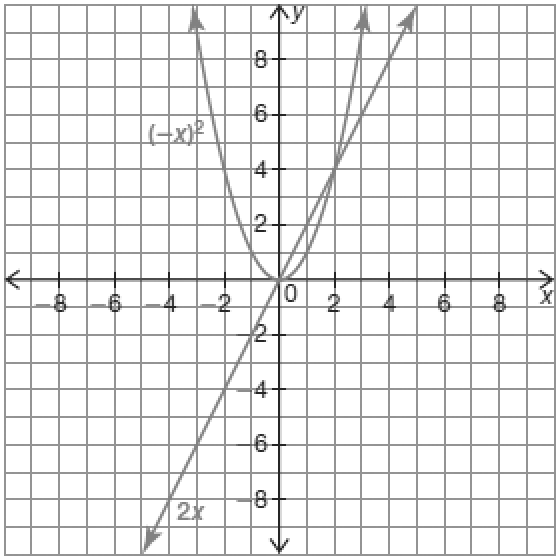
I eliminated this function because the graph represents a function with 4 roots and this function only has 2.

I eliminated this function because the graph represents an even function with a positive *a*-value and this function is odd degree with a negative *a*-value.

36. The cubic function will increase from left to right, so the sketch may show either all positive functions or two negative. All *x*-intercepts are at 0 because the cubic function needs to pass through the origin. The following functions representing one possible solution are shown.

$f(x) = (-x)^2$

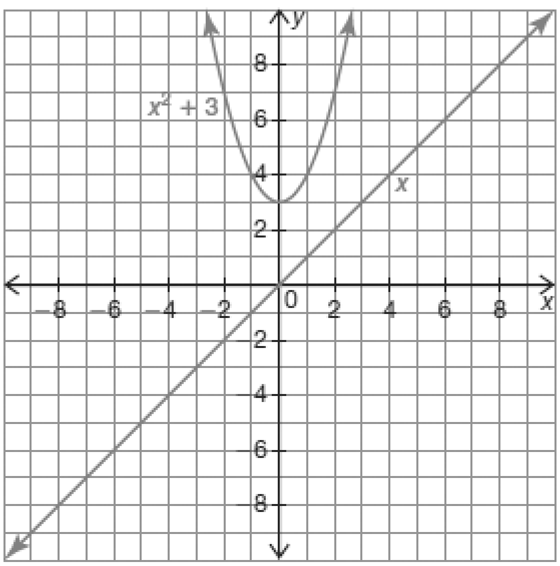
$g(x) = 2x$



37. The graph must be a linear function with a real root $x = 0$ and a quadratic function above the *x*-axis because the roots must be imaginary. The following functions representing one possible solution are shown.

$f(x) = x$

$g(x) = x^2 + 3$



38. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2) - f(1)}{2 - 1}$

$= \dfrac{6 - 0}{1}$

$= 6$

39.

$$\begin{array}{r|rrrrr} -2 & 1 & -10 & 8 & 106 & -105 \\ & & -2 & 24 & -64 & -84 \\ \hline & 1 & -12 & 32 & 42 & \boxed{-189} \end{array}$$

When $p(x)$ is divided by $x+2$, the remainder is -189. So, by the Remainder Theorem $p(-2) = -189$.

40.

$$\begin{array}{r|rrrrr} -3 & 2 & 5 & 8 & 15 & 6 \\ & & -6 & 3 & -33 & 54 \\ \hline & 2 & -1 & 11 & -18 & \boxed{60} \end{array}$$

When $p(x)$ is divided by $x+3$, the remainder is 60. So, by the Remainder Theorem $p(-3) = 60$.

41.

$$\begin{array}{r l} & x^3 + 4x^2 - 2x - 10 \\ x - 1 \overline{)} & x^4 + 3x^3 - 6x^2 - 8x + 0 \\ & \underline{x^4 - x^3} \\ & \quad 4x^3 - 6x^2 \\ & \quad \underline{4x^3 - 4x^2} \\ & \qquad -2x^2 - 8x \\ & \qquad \underline{-2x^2 + 2x} \\ & \qquad\quad -10x + 0 \\ & \qquad\quad \underline{-10x + 10} \\ & \qquad\qquad -10 \end{array}$$

When $p(x)$ is divided by $x-1$, the remainder is -10. So, by the Remainder Theorem $p(1) = -10$.

42. If $x-3$ is a factor of $f(x)$, then by the Factor Theorem $f(3) = 0$.

$$f(3) = 4(3)^4 - (3)^3 - 52(3)^2 - 35(3) + 12$$
$$f(3) = 324 - 27 - 468 - 105 + 12$$
$$f(3) = -264$$

When $f(x)$ is evaluated at 3, the result is -264. According to the Factor Theorem $x-3$ is not a factor of $f(x)$.

43. If $3x+4$ is a factor of $f(x)$, then by the Factor Theorem $f\left(-\frac{4}{3}\right) = 0$.

$$f\left(-\frac{4}{3}\right)=3\left(-\frac{4}{3}\right)^3+13\left(-\frac{4}{3}\right)^2+18\left(-\frac{4}{3}\right)+8$$

$$f\left(-\frac{4}{3}\right)=-\frac{64}{9}+\frac{208}{9}-24+8$$

$$f\left(-\frac{4}{3}\right)=0$$

When $f(x)$ is evaluated at $-\frac{4}{3}$, the result is 0. According to the Factor Theorem $3x+4$ is a factor of $f(x)$.

44. $f(2)=(2)^3+8(2)^2-11(2)-18$

$f(2)=8+32-22-18$

$f(2)=0$

$f(-1)=(-1)^3+8(-1)^2-11(-1)-18$

$f(-1)=-1+8+11-18$

$f(-1)=0$

$f(-9)=(-9)^3+8(-9)^2-11(-9)-18$

$f(-9)=729+648+99-18$

$f(-9)=0$

Yes, the function $g(x)$ is the factored form of $f(x)$. When each of the zeros of the factors of $g(x)$ is evaluated in $f(x)$, the result is 0.

45. $x^2-16x-36=(x+2)(x-18)$

46. $x^3+x^2-4x-4=x^2(x+1)-4(x+1)$

$$=(x^2-4)(x+1)$$

$$=(x+2)(x-2)(x+1)$$

47. $x^4-3x^3-x^2-3x=x^3(x-3)-x(x-3)$

$$=(x^3-x)(x-3)$$

$$=x(x^2-1)(x-3)$$

$$=x(x-1)(x+1)(x-3)$$

48. $x^4-29x^2+100=(x^2-25)(x^2-4)$

$$=(x-5)(x+5)(x-2)(x+2)$$

49. $a^3-b^3=(a-b)(a^2+ab+b^2)$

$x^3-8y^3=(x)^3-(2y)^3$

$$=(x-2y)(x^2+2xy+4y^2)$$

50. $a^3+b^3=(a+b)(a^2-ab+b^2)$

$x^3+64y^3=(x)^3+(4y)^3$

$$=(x+4y)(x^2-4xy+16y^2)$$

51. $a^2 - b^2 = (a+b)(a-b).$

$$\begin{aligned} x^4 - 36 &= (x^2)^2 - (6)^2 \\ &= (x^2+6)(x^2-6) \\ &= (x^2+6)(x-\sqrt{6})(x+\sqrt{6}) \end{aligned}$$

52. $a^2 - b^2 = (a+b)(a-b).$

$$\begin{aligned} 49x^2 - 4y^2 &= (7x)^2 - (2y)^2 \\ &= (7x-2y)(7x+2y) \end{aligned}$$

53. $a^2 + 2ab + b^2 = (a+b)^2$

$$\begin{aligned} 9x^4 + 42x^2y + 49y^2 &= (3x^2)^2 + 2(3x^2)(7y) + (7y)^2 \\ &= (3x^2+7y)^2 \end{aligned}$$

54. $8x^4 - 16x^3 + 56x^2 - 24x = 8x(x^3 - 2x^2 + 7x - 3)$

55. $25x^2 - 35x + 12 = (5x)^2 - 7(5x) + 12$

Let $z = 5x$

$$\begin{aligned} &= z^2 - 7z + 12 \\ &= (z-3)(z-4) \\ &= (5x-3)(5x-4) \end{aligned}$$

56.
- Possible rational roots:

$p = \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

$q = \pm 1$

$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

- Solve completely:

$$\begin{array}{r|rrrr} 4 & 1 & 3 & -18 & -40 \\ & \downarrow & 4 & 28 & 40 \\ \hline & 1 & 7 & 10 & \boxed{0} \end{array}$$

$$\begin{aligned} x^3 + 3x^2 - 18x - 40 &= (x-4)(x^2+7x+10) \\ &= (x-4)(x+2)(x+5) \end{aligned}$$

$x = 4, -2, -5$

57.
- Possible rational roots:

$p = \pm 1,\ \pm 17$

$q = \pm 1$

$\frac{p}{q} = \pm 1,\ \pm 17$

- Solve completely:

There are no rational roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(17)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{-4}}{2}$$

$$= 4 + i,\ 4 - i$$

$$x = 4 + i,\ 4 - i$$

58. $\frac{3}{x} + \frac{1}{x+1} = \frac{3(x+1)}{x(x+1)} + \frac{1(x)}{(x+1)(x)}$

$$= \frac{3x+3}{x(x+1)} + \frac{x}{x(x+1)}$$

$$= \frac{4x+3}{x(x+1)};\ x \neq -1,\ 0$$

59. $\frac{1}{x+3} - \frac{1}{x-3} = \frac{1(x-3)}{(x+3)(x-3)} - \frac{1(x+3)}{(x-3)(x+3)}$

$$= \frac{x-3}{(x+3)(x-3)} - \frac{x+3}{(x+3)(x-3)}$$

$$= \frac{-6}{(x+3)(x-3)};\ x \neq \pm 3$$

60. $\frac{x+1}{x^2-16} - \frac{x}{x^2+7x+12} = \frac{x+1}{(x-4)(x+4)} - \frac{x}{(x+3)(x+4)}$

$$= \frac{(x+1)(x+3)}{(x-4)(x+4)(x+3)} - \frac{x(x-4)}{(x+3)(x+4)(x-4)}$$

$$= \frac{x^2+4x+3}{(x-4)(x+4)(x+3)} - \frac{x^2-4x}{(x+3)(x+4)(x-4)}$$

$$= \frac{8x+3}{(x-4)(x+4)(x+3)};\ x \neq -4,\ -3,\ 4$$

61. Restrictions: $x \neq -2,\ \frac{1}{2},\ 4$

$$\begin{aligned}
\frac{1}{2x^2+3x-2}\cdot\frac{x^2-2x-8}{x-4} &= \frac{1}{(x+2)(2x-1)}\cdot\frac{(x+2)(x-4)}{x-4}\\
&= \frac{1}{\cancel{(x+2)}(2x-1)}\cdot\frac{\cancel{(x+2)}\cancel{(x-4)}}{\cancel{x-4}}\\
&= \frac{1}{2x-1}
\end{aligned}$$

62. Restrictions: $x \neq -4, -\frac{2}{3}, 1$

$$\begin{aligned}
\frac{x^2+6x+8}{3x+2} \div \frac{-x-4}{3x^2-x-2} &= \frac{x^2+6x+8}{3x+2}\cdot\frac{3x^2-x-2}{-x-4}\\
&= \frac{(x+4)(x+2)}{3x+2}\cdot\frac{(3x+2)(x-1)}{-1(x+4)}\\
&= \frac{\cancel{(x+4)}(x+2)}{\cancel{3x+2}}\cdot\frac{\cancel{(3x+2)}(x-1)}{-1\cancel{(x+4)}}\\
&= \frac{x^2+x-2}{-1}\\
&= -x^2-x+2
\end{aligned}$$

63. Restrictions: $x \neq 0, 3$

$$\begin{aligned}
\frac{9}{x-3} &= \frac{27}{x(x-3)}+\frac{6}{x}\\
x(x-3)\left[\frac{9}{x-3}\right] &= x(x-3)\left[\frac{27}{x(x-3)}+\frac{6}{x}\right]\\
9x &= 27+6x-18\\
3x &= 9\\
x &= 3
\end{aligned}$$

However, $x \neq 3$ because it is a restriction on the variable and thus is an extraneous root. This equation has no solution.

64.

Restrictions: $x \neq -1, 0, 1$

$$\begin{aligned}
(x-3)(x^2-1) &= x^2(x-3)\\
x^3-3x^2-x+3 &= x^3-3x^2\\
-x+3 &= 0\\
-x &= -3\\
x &= 3
\end{aligned}$$

Check $x = 3$.

$$\begin{aligned}
\frac{3-3}{(3)^2} &\stackrel{?}{=} \frac{3-3}{(3)^2-1}\\
\frac{0}{9} &\stackrel{?}{=} \frac{0}{8}\\
0 &= 0 \quad \checkmark
\end{aligned}$$

65.

Check $x = 2$.

Restriction: $x \neq 0$

$$2x^2\left(\frac{2}{x} - \frac{3}{2x}\right) = 2x^2\left(\frac{1}{x^2}\right)$$

$$4x - 3x = 2$$

$$x = 2$$

$$\frac{2}{2} - \frac{3}{2(2)} \stackrel{?}{=} \frac{1}{2^2}$$

$$1 - \frac{3}{4} \stackrel{?}{=} \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4} \quad \checkmark$$

66. Restrictions: $x \neq -2, -\frac{3}{2}, -1$

$$\frac{x}{x+2} + \frac{2(2x+3)}{(x+1)(2x+3)} = \frac{x-1}{2(x+2)}$$

$$\frac{x}{x+2} + \frac{2}{(x+1)} = \frac{x-1}{2(x+2)}$$

$$2(x+1)(x+2)\left[\frac{x}{x+2} + \frac{2}{(x+1)}\right] = 2(x+1)(x+2)\left[\frac{x-1}{2(x+2)}\right]$$

$$2(x+1)x + 2(x+2)(2) = (x+1)(x-1)$$

$$2x^2 + 6x + 8 = x^2 - 1$$

$$x^2 + 6x + 9 = 0$$

$$(x+3)(x+3) = 0$$

$$x = -3$$

Check $x = -3$.

$$\frac{-3}{-3+2} + \frac{4(-3)+6}{2(-3)^2 + 5(-3) + 3} \stackrel{?}{=} \frac{-3-1}{2(-3)+4}$$

$$\frac{-3}{-1} + \frac{-6}{6} \stackrel{?}{=} \frac{-4}{-2}$$

$$2 = 2 \quad \checkmark$$

67. Hector should add 4 quarts of water to obtain a drink that is 20% orange concentrate.

Let *x* represent the number of quarts of water needed.

$$\frac{0.6(2)}{2+x} = 0.2;\ \text{Restriction: } x \neq -2$$

$$\frac{1.2}{2+x} = 0.2$$

$$1.2 = 0.4 + 0.2x$$

$$0.8 = 0.2x$$

$$x = 4$$

68. Working by himself, it will take Benny 4 hours to wash 24 golf carts.

Let *x* represent the number of hours it will take Benny to wash 24 golf carts by himself.

Portion of the golf carts Benny can wash $= 2\left(\frac{1}{x}\right) = \frac{2}{x}$

Portion of the golf carts Kendall can wash $= 2\left(\frac{1}{4}\right) = \frac{2}{4}$

$$\frac{2}{4} + \frac{2}{x} = 1;\ \text{Restriction: } x \neq 0$$

$$4x\left(\frac{2}{4} + \frac{2}{x}\right) = 4x(1)$$

$$2x + 8 = 4x$$

$$8 = 2x$$

$$x = 4\,hours$$

69. Working together, it will take Nyesha and Shantese $27\frac{3}{11}$ hours to service all of their customers.

Let *x* represent the number of hours it will take to service all of their customers while working together.

$$\frac{x}{50} + \frac{x}{60} = 1$$

$$300\left(\frac{x}{50} + \frac{x}{60}\right) = 300(1)$$

$$6x + 5x = 300$$

$$11x = 300$$

$$x = \frac{300}{11} \text{ or } 27\frac{3}{11}\,hours$$

70. $f^{-1}(x) = \pm\frac{1}{2}\sqrt{x+1}$

71. $f^{-1}(x) = \sqrt[3]{\frac{x-2}{3}}$

72. $f^{-1}(x) = \pm\sqrt[4]{x+16} - 2$

73. $f^{-1}(x) = \left(\frac{x+1}{2}\right)^5$

Domain: all real numbers

Range: all real numbers

74. $= (16x^{10}y^8z^2)^{\frac{1}{2}} = 4y^4\left|x^5z\right|$

75. $= 5x + 15x - 10x = 10x$

76. $= 25x(8\sqrt{x}) = 200x\sqrt{x}$

77. $= \dfrac{5(2)^{\frac{1}{2}}y|x|}{3(2)^{\frac{5}{4}}x^{\frac{1}{2}}y^{\frac{5}{4}}} = \dfrac{5x^{\frac{2}{4}}}{3(2)^{\frac{3}{4}}y^{\frac{1}{4}}} = \dfrac{5}{3}\cdot\sqrt[4]{\dfrac{x^2}{8y}}$

y must be positive because of $\sqrt[4]{y^5}$.

78. $= 9\sqrt{x} + \sqrt[4]{x}$

79.

$$4x^2 - 8x + 4 = x + 2$$
$$4x^2 - 9x + 2 = 0$$
$$(4x-1)(x-2) = 0$$
$$x = \frac{1}{4},\ x = 2$$

Solution: $x = 2$

$$2\left(\frac{1}{4}\right) - 2 \overset{?}{=} \sqrt{\left(\frac{1}{4}\right) + 2}$$
$$\frac{-3}{2} \overset{?}{=} \sqrt{\frac{9}{4}}$$
$$\frac{-3}{2} \neq \frac{3}{2}$$

Extraneous Root

$$2(2) - 2 \overset{?}{=} \sqrt{(2) + 2}$$
$$2 \overset{?}{=} 4$$
$$2 = 2$$

80. $(\sqrt[5]{3x-3})^5 = (2)^5$

$$3x - 3 = 32$$
$$3x = 35$$
$$x = \frac{35}{3}$$

81. $\sqrt{9x+3} = 3$

$$(\sqrt{9x+3})^2 = (3)^2$$
$$9x + 3 = 9$$
$$9x = 6$$
$$x = \frac{2}{3}$$

82. $\dfrac{-8x^3y^{12}}{x^8y^{12}} = \dfrac{8}{x^5}$

83. $\left(\dfrac{2y^3}{x^2}\right)^{-2} = \left(\dfrac{x^2}{2y^3}\right)^2 = \dfrac{x^4}{4y^6}$

84. $\dfrac{x^3x^4}{y^5y^2} = \dfrac{x^7}{y^7}$

85. 4

86. 5

87. The log base 4 of 6 is *x*.

$4^x = 6$

88. $\log_5 10 + 9\log_5 x + 7\log_5 y$

89. $\log_3 2 - 2\log_3 x - 10\log_3 y$

90. $\log_3 6x^5$

91. $\log_7 122x^5y^8$

92. $\dfrac{7\pi}{4} \times \dfrac{360°}{2\pi} = 315°$

93. $\dfrac{11\pi}{6} \times \dfrac{360°}{2\pi} = 330°$

94. $240° \times \dfrac{2\pi}{360°} = \dfrac{4\pi}{3}$

95. $135° \times \dfrac{2\pi}{360°} = \dfrac{3\pi}{4}$

96. $\sin\dfrac{5\pi}{4} = -\dfrac{\sqrt{2}}{2}$

97. $\cos\dfrac{2\pi}{3} = -\dfrac{1}{2}$

98. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{8}{17} \div \dfrac{15}{17} = \dfrac{8}{17} \times \dfrac{17}{15} = \dfrac{8}{15}$

99. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{2\sqrt{5}}{5} \div \dfrac{\sqrt{5}}{5} = \dfrac{2\sqrt{5}}{5} \times \dfrac{5}{\sqrt{5}} = 2$

100. $\tan\dfrac{5\pi}{4} = -\dfrac{\sqrt{2}}{2} \div -\dfrac{\sqrt{2}}{2} = 1$

101. $\theta = 120°, 300°$

102. $\theta = 45°$

103. $\theta = 240°$

104. $x = \dfrac{5\pi}{6}$ radians, $\dfrac{7\pi}{6}$ radians

105. $x = 0$ radians, π radians, 2π radians

106. amplitude = 3

107. amplitude = $\dfrac{1}{2}$

108. 0.64, 0.39, 0.14, -0.11, -0.36, -0.61

109. 4, 4.5, 5, 5.5

110.

$$\begin{array}{r} 2x^2 + x - 15 \\ x-4\,\big)\overline{2x^3 - 7x^2 - 19x + 60} \\ \underline{2x^3 - 8x^2} \qquad\qquad \\ x^2 - 19x \qquad \\ \underline{x^2 - \ \ 4x} \qquad \\ -15x + 60 \\ \underline{-15x + 60} \\ 0 \end{array}$$

$$2x^3 - 7x^2 - 19x + 60 = (x-4)(2x^2 + x - 15)$$

111.

$$\begin{array}{r} 2x^2 + 3x - 7 \\ x-2\,\big)\overline{2x^3 - \ x^2 - 13x - 6} \\ \underline{2x^3 + 4x^2} \qquad\qquad \\ 3x^2 - 13x \qquad \\ \underline{3x^2 - \ \ 6x} \qquad \\ -7x - \ \ 6 \\ \underline{-7x + 14} \\ -20 \end{array}$$

$$2x^3 - x^2 - 13x - 6 = (x-2)\left((2x^2 + 3x - 7) + \frac{-20}{x-2}\right)$$

112. $p = \pm 1,\ \pm 3,\ \pm 5,\ \pm 15$

$q = \pm 1,\ \pm 2$

$$\frac{p}{q} = \pm 1,\ \pm 3,\ \pm 5,\ \pm 15,\ \pm \frac{1}{2},\ \pm \frac{3}{2},\ \pm \frac{5}{2},\ \pm \frac{15}{2}$$

113. $p = \pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 9,\ \pm 18$

$q = \pm 1,\ \pm 2$

$$\frac{p}{q} = \pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 9,\ \pm 18,\ \pm \frac{1}{2},\ \pm \frac{3}{2},\ \pm \frac{9}{2}$$

114. $r = \frac{2}{7}$

The series is convergent because the common ratio is between 0 and 1.

115. $r = 0.1$

The series is convergent because the common ratio is between 0 and 1.

116. The first term, g_1, is \$1500 and represents Panna's initial investment in her 401K plan. The common ratio, *r*, is 0.985 and represents the factor by which her previous investment will decrease each year.

$$g_n = 1500 \cdot 0.985^{n-1}$$

117. I can use Euclid's Method, $S_n = \dfrac{g_n(r) - g_1}{r - 1}$, where $n = 10$, $g_{10} = 786{,}432$, $r = 4$, and $g_1 = 3$, to determine how many people have the flu during the 10-week period.

$$\begin{aligned} S_{10} &= \frac{g_{10}(r) - g_1}{r - 1} \\ &= \frac{786{,}432(4) - 3}{4 - 1} \\ &= 1{,}048{,}575 \end{aligned}$$

The total number of people who will have the flu during the 10-week period is 1,048,575 people.

118. $\log_{15}(x^2 - 2x) = 1$

$$\begin{aligned} 15^1 &= x^2 - 2x \\ 0 &= x^2 - 2x - 15 \\ 0 &= (x + 3)(x - 5) \\ x &= -3, 5 \end{aligned}$$

Check:

$$\begin{aligned} \log_{15}((-3)^2 - 2(-3)) &\stackrel{?}{=} 1 \\ \log_{15}(9 + 6) &\stackrel{?}{=} 1 \\ \log_{15} 15 &= 1 \end{aligned}$$

$$\begin{aligned} \log_{15}(5^2 - 2(5)) &\stackrel{?}{=} 1 \\ \log_{15}(25 - 10) &\stackrel{?}{=} 1 \\ \log_{15} 15 &= 1 \end{aligned}$$

119. $2\log_3 x - \log_3 8 = \log_3(x-2)$

$$\log_3 x^2 - \log_3 8 = \log_3(x-2)$$

$$\log_3\left(\frac{x^2}{8}\right) = \log_3(x-2)$$

$$\frac{x^2}{8} = x-2$$

$$x^2 = 8x-16$$

$$x^2 - 8x + 16 = 0$$

$$(x-4)^2 = 0$$

$$x = 4$$

Check:

$$2\log_3 4 - \log_3 8 \stackrel{?}{=} \log_3(4-2)$$

$$\log_3 16 - \log_3 8 \stackrel{?}{=} \log_3 2$$

$$\log_3\left(\frac{16}{8}\right) \stackrel{?}{=} \log_3 2$$

$$\log_3 2 = \log_3 2$$

120. $\log_4(x+3) + \log_4 x = 1$

$$\log_4(x(x+3)) = 1$$

$$x(x+3) = 4^1$$

$$x^2 + 3x = 4$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x = -4, 1$$

Check:

−4 is an extraneous solution.

$$\log_4(1+3) + \log_4 1 \stackrel{?}{=} 1$$

$$\log_4 4 + \log_4 1 \stackrel{?}{=} 1$$

$$1 + 0 \stackrel{?}{=} 1$$

$$1 = 1$$

121. Answers may vary

122. C