2.1B Vertex Form of a Quadratic Function

Objectives:

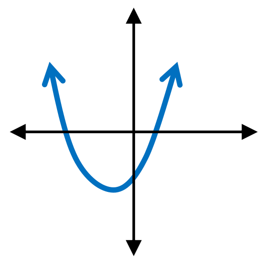
F.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context.

F.IF.7: Graph functions expressed symbolically and show key features of the graph: intercepts, maxima, and minima.

Anticipatory Set:

A **quadratic function** is a U shaped graph called a **parabola**

It can be represented using two different forms.



1. **vertex form**: $f(x) = a(x – h)^2 + k$ where $a \neq 0$
2. **standard form**: $f(x) = ax^2 + bc + c$ where $a \neq 0$

All quadratics have certain characteristics.

x-intercepts (zeros): Points where the parabola crosses the x axis.

y-intercept: Point where the parabola crosses the y-axis.

Axis of Symmetry: Vertical line (x = number) which divides the parabola into mirror image halves.

Vertex: Maximum/minimum point. Found on the axis of symmetry.

Concavity: Does the parabola open up or down?

Instruction:

Vertex form is also known as the **graphing form** because you can easily graph a function from it using transformations to the function $f(x) = x^2$.

The **vertex form** of a quadratic function is $f(x) = a(x – h)^2 + k$, where a, h, and k are constants.

Concavity:

If the "a" is positive ($a > 0$) then the parabola opens **upward**, and it has a **minimum,** lowest point.

If the "a" is negative ($a < 0$) then the parabola opens **downward**, and it has a **maximum,** highest point.

If $a > 1$, then the parabola is vertically stretched by a factor of a.

If $0 < a < 1$, then the parabola is vertically compressed by a factor of a.

Example: Determine the concavity of the function $f(x) = 3(x – 4)^2 + 5$.
Does this function have a minimum or a maximum?

Since a = 3 and 3 is positive, the function opens up and has a minimum.

White Board Activity:

Practice: Determine the concavity each of the functions. Determine whether the function has a maximum or a minimum.

a. $f(x) = -2(x + 6)^2 – 4$
opens down
maximum

b. $f(x) = ½ (x – 9)^2 + 9$
opens up
minimum

The **axis of symmetry** is the vertical line through the vertex of the parabola that divides the parabola into two congruent halves.

The function $f(x) = x^2$ has axis of symmetry x = 0 (the y-axis).

If the function is reflected across the x-axis or the y-axis then the axis of symmetry is not effected and will remain x = 0.

If the function is stretched or compressed either vertically or horizontally then the axis of symmetry is not effected and will remain x = 0.

If the function is shifted up or down then the axis of symmetry will not be affected and will remain x = 0.

If the function is shifted right or left then the axis of symmetry will also be shifted right or left.

The right/left shift then determines the axis of symmetry. It has equation **x = h**.

Open the book to page 67 and read example 1.

Example: Identify the axis of symmetry for the graph of $f(x) = -\frac{1}{2}(x + 5)^2 - 8$.

Note: This function is compressed by a factor of ½ and reflected over the x-axis (which have no effect).

This is a shift down 8 (which has no effect).

This is a shift left 5 (which does have an effect).

axis of symmetry: x = -5

White Board Activity:

Practice 1: Identify the axis of symmetry for the graph of $f(x) = (x - 3)^2 + 1$.

Axis of symmetry: x = 3

The **vertex** of the function $f(x) = x^2$ is (0, 0).

If the function is stretched or compressed or reflected over the x-axis or y-axis, then the vertex is not effected and will remain (0, 0).

If the function is shifted right/left and/or up/down, then the vertex will also be shifted to match.

The right/left and up/down shift then determine the vertex.

Vertex:

The lowest (**minimum**) or highest (**maximum**) point of the parabola

h indicates a horizontal translation from (0, 0).

k indicates a vertical translation from (0, 0).

These translations determine the ordered pair, **(h, k).**

Example: What is the vertex of the function $f(x) = 3(x – 4)^2 + 5$. Is this point a minimum or maximum?

V(4, 5), minimum

White Board Activity:

Practice: What is the vertex of each of the functions below. Is it a maximum or a minimum.

a. $f(x) = -2(x + 6)^2 – 4$ — **V(-6, -4), maximum**

b. $f(x) = \frac{1}{2}(x – 9)^2 + 9$ — **V(9, 9), minimum**

Domain and Range:

The **domain** of a function is the set of all the possible inputs (replacements for x).

Since any number can replace the x, the domain of a quadratic function is “All real numbers.”

The **range** of a function is the set of all the possible outputs (replacements for y).

The range depends on whether "a" is positive or negative and the value of k.

If $a < 0$, then the function opens down and has a maximum so the range is the set of all y's that are less than or equal to the vertex y value, k. **$\{y | y \leq k\}$**

If $a > 0$, then the function opens up and has a minimum so the range is the set of all y's that are greater than or equal to the vertex y value, k. **$\{y | y \geq k\}$**

Example: State the domain and range of the function $f(x) = 3(x - 4)^2 + 5$.

Domain: All real numbers.

Since the value of a is positive (a = 3) the graph opens up and has a minimum at the vertex.

Range: $\{y | y \geq 5\}$

White Board Activity:

Practice: State the domain and range of each function.

a. $f(x) = -2(x + 6)^2 - 4$

domain: all real numbers

range: $\{y | y \leq -4\}$

b. $f(x) = ½ (x - 9)^2 + 9$

domain: all real numbers

range: $\{y | y \geq 0\}$

y-intercept:

Find f(0) or let x = 0 and find y. Plot the ordered pair on the y-axis.

Example: Find the y-intercept of the function $f(x) = 3(x - 4)^2 + 5$.

$f(0) = 3(0 - 4)2 + 5 = 3(16) + 5 = 53$ (0, 53)

White Board Activity:

Practice: Find the y-intercept of each function.

a. $f(x) = -2(x + 6)^2 - 4$

$f(0) = -2(0 + 6)^2 - 4$

$= -76$

(0, -76)

b. $f(x) = ½ (x - 9)^2 + 9$

$f(0) = ½ (0 - 9)^2 + 9$

$= 49.5$

(0, 49.5)

Paper/Pencil Graphing:

Example: Graph $f(x) = -(x - 3)^2 + 2$.

Steps:

1. What is the concavity? Does the function open up or down? **down**

2. Does the function have a maximum or a minimum? **maximum**

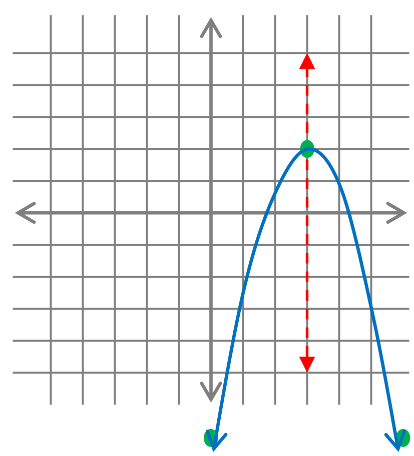
3. What is the equation of the axis of symmetry? Draw the line. **x = 3**

3. What is the vertex. Plot the point. **(3, 2)**

5. What is the domain and range? **$\{y | y \leq 2\}$**

6. What is the y-intercept. Plot the point. **(0, -7)**

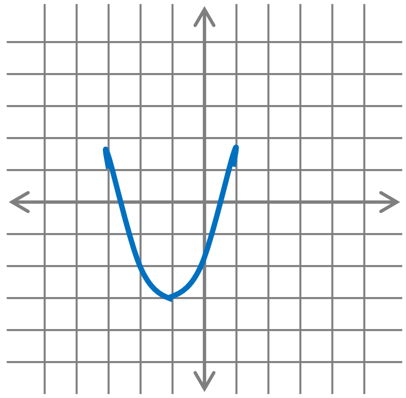
7. Using the axis of symmetry plot the reflection of the y-intercept. **(6, -7)**

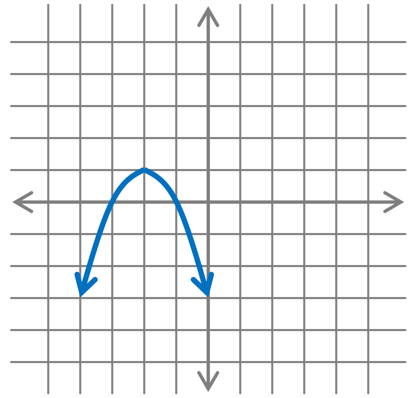


Practice: Graph each of the following.

a. $f(x) = (x + 1)^2 - 3$

b. $f(x) = -(x - 2)^2 + 1$





Assessment:

Question student pairs.

Independent Practice:

Handout 2.1B

For a Grade:

Assignment 2.1B