2.2A Properties of Quadratic Functions in Standard Form

Objectives:

F.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context.

F.IF.7: Graph functions expressed symbolically and show key features of the graph: intercepts, maxima, and minima.

For the Board:

You will be able to define, identify, and graph quadratic functions.

You will be able to identify and use maximums and minimums of quadratic functions to solve problems.

Bell Work 2.2:

1. Given $f(x) = -(x - 2)^2 + 3$ answer each of the following.
    a. Does the function open up or down?
    b. Does it have a maximum or a minimum?
    c. What is the axis of symmetry?
    d. What is the vertex?
    e. What is the y-intercept?
    f. What is the domain?
    g. What is the range?

Anticipatory Set:

The **vertex form** of a quadratic equation has both advantages and disadvantages.

- It is easy to determine concavity, axis of symmetry, and vertex.
- It requires only a small amount of arithmetic to determine the y-intercept.
- It is easy to get an accurate paper-pencil graph.
- It is difficult to determine the x-intercepts (zeros) of the function.

Unfortunately, most applications of quadratics involve finding the x-intercepts (zeros) of the function. For this reason, a second form of the quadratic is necessary.

Instruction:

Quadratic functions can also be represented in **standard form**: $f(x) = ax^2 + bx + c$ where $a \neq 0$.

Example: Convert each of the following quadratics in vertex form to quadratics in standard form.

1. $f(x) = (x + 2)^2 - 3$

   $f(x) = (x + 2)(x + 2) - 3$

   $f(x) = x^2 + 4x + 4 - 3$

   $f(x) = x^2 + 4x + 1$

2. $f(x) = 3(x - 1)^2 + 4$

   $f(x) = 3(x - 1)(x - 1) + 4$

   $f(x) = 3(x^2 - 2x + 1) + 4$

   $f(x) = 3x^2 - 6x + 3 + 4$

   $f(x) = 3x^2 - 6x + 7$

White Board Activity:
Practice: Convert the function $f(x) = -2(x + 3)^2 + 8$ into standard form.

$f(x) = -2(x + 3)(x + 3) + 8 = -2(x^2 + 6x + 9) + 8 = -2x^2 - 12x - 18 + 8 = -2x^2 - 12x - 10$

Converting standard form to vertex form is more difficult and requires that we study some additional algebra first.

The coefficients a, b, and c are used to determine the properties of the quadratic when it is in standard form.

Concavity:

The **a** value determines concavity and therefore whether the function has a maximum or a minimum.

a < 0: opens down, vertex is a maximum

a > 0: opens up, vertex is a minimum

Example: Determine whether the function $f(x) = -3x^2 + 4x + 5$ opens up or down.
Does the function have a maximum or minimum.

a = -3 so the function opens down and has a maximum.

White Board Activity:
Practice: Determine whether each function opens up or down. Does it have a maximum or a minimum.

a. $f(x) = 2x^2 + 5x + 9$
a = 2 so the function opens up and has a minimum

b. $f(x) = -x^2 - 3x + 10$
a = -1 so the function opens down and has a maximum.

Axis of Symmetry: The axis of symmetry is a vertical line with equation $x = \dfrac{-b}{2a}$.

Example: What is the equation of the axis of symmetry of the function $f(x) = -3x^2 + 4x + 5$?

a = -3 and b = 4 so x = -b/2a becomes x = -4/2(-3) or x = 2/3.

White Board Activity:
Practice: What is the equation of the axis of symmetry of each of the following functions?

a. $f(x) = 2x^2 + 5x + 9$
a = 2 and b = 5 so
x = -b/2a is x = -5/2(2)
x = -5/4

b. $f(x) = -x^2 - 3x + 10$
a = -1 and b = -3 so
x = -b/2a is x = -(-3)/2(-1)
x = -3/2

Vertex: $\left(\dfrac{-b}{2a}, c - \dfrac{b^2}{4a}\right)$ or use the $\dfrac{-b}{2a}$ as x in the function and solve for f(x).

Example: What is the vertex of the function $f(x) = -3x^2 + 4x + 5$?
From the example problem above x = -b/2a = 2/3.

$f(2/3) = -3(2/3)^2 + 4(2/3) + 5 = -3(4/9) + 8/3 + 5 = -4/3 + 8/3 + 15/3 = 19/3$

Vertex: (2/3, 19/3) is a maximum

White Board Activity:

Practice: What is the vertex of each of the following functions? Is it a maximum or a minimum?

a. $f(x) = 2x^2 + 5x + 9$

$x = -5/4 = -1.25$

$f(-5/4) = 2(-1.25)^2 + 5(-1.25) + 9$

$= 5.875$

Vertex: (-1.25, 5.875)

is a minimum.

b. $f(x) = -x^2 - 3x + 10$

$x = -3/2 = -1.5$

$f(-3/2) = -(-1.5)^2 - 3(-1.5) + 10$

$= 12.25$

Vertex: (-1.5, 12.25)

is a maximum.

Open the book to page 71 and read example 4.

Example: The height h in meters of a weather balloon can be modeled by the function $h(t) = -0.024t^2 + 1.28t + 33.6$, where t is the time in minutes after the balloon was launched. Based on this model, what is the maximum height reached by the balloon and when does the balloon reach this height?

What are you being asked to find? Look for key words:

"maximum": The maximum occurs at the vertex, so find the vertex.

Time will be the x-value of the vertex and height will be the y-value of the vertex.

$-b/2a = -(1.28)/2(-0.024) = 26.7$ rounded to the nearest tenth.

$f(26.7) = -0.024(26.7)^2 + 1.28(26.7) + 33.6 = 66.1$ rounded to the nearest tenth.

The maximum height of the balloon will be 66.1 m and will occur 26.7 minutes after launch.

White Board Activity:

Practice: The highway mileage m in miles per gallon for a compact car is approximated by $m(s) = -0.025s^2 + 2.45s - 30$, where s is the speed in miles per hour. What is the maximum mileage for this compact car to the nearest tenth of a mile per gallon? What speed results in this mileage?

$-b/2a = -2.45/2(-0.025) = 49$

$f(49) = -0.025(49)2 + 2.45(49) - 30 = 30.0$

The maximum is 49 miles per gallon occurring at a speed of 30 miles per hour.

The **domain** of a function is the set of all possible inputs.

All quadratics have a domain of "all real numbers".

Applications problems may have domains which are more limited due to the conditions of the problem.

Example: It may not be reasonable for time or speed to be negative.

The **range** of a function is the set of all possible outputs.

Application problems may have ranges which are more limited due to the conditions of the problem.

Example: It may not be reasonable for height or mileage to be negative.

If a quadratic function opens up, "a" is positive, then it has a minimum and the range is $f(x) \geq c - b^2/4a$. Written: $\{y \mid y \geq c - b^2/4a\}$ or $[c - b^2/4a, \infty)$.

If a quadratic function opens down "a" is negative, then it has a maximum and the range is $f(x) \leq c - b^2/4a$. Written: $\{y \mid y \leq c - b^2/4a\}$ or $(-\infty, c - b^2/4a]$.

Example: What is the domain and range of the function $f(x) = -3x^2 + 4x + 5$?

Domain: All real numbers.
Since the "a" is negative, then graph opens down and has a maximum.
The x value of the vertex is 2/3 so the y value of the vertex is 19/3.
Range: $\{y | y \leq 19/3\}$ or $(-\infty, 19/3]$.

White Board Activity:
Practice: What is the domain and range of each of the following functions?

a. $f(x) = 2x^2 + 5x + 9$
domain: all real numbers
range: $\{y | y \geq 5.875\}$
$[5.875, \infty)$

b. $f(x) = -x^2 - 3x + 10$
domain: all real numbers
range: $\{y | y \leq 12.25\}$
$(-\infty, 12.25]$

Investigate the y-intercept:
The **y-intercept** of a function is the point where the graph crosses the y-axis.
Let x = 0 and solve for y or find f(0).
Consider each of the following. Find the y-intercept. (Do f(0).)
What do you notice about the y-intercept?

a. $f(x) = 3x^2 - 4x + 2$ **$f(0) = 2$**
b. $f(x) = x^2 - 7$ **$f(0) = -7$**
c. $f(x) = -4x^2 + 2x - 9$ **$f(0) = -9$**

When the equation is in standard form, the y-intercept is **(0, c)**.

Example: What is the y-intercept of the function $f(x) = -3x^2 + 4x + 5$?

y-intercept: (0, 5)

White Board Activity:
Practice: What is the y-intercept of each of the following functions?

a. $f(x) = 2x^2 + 5x + 9$
(0, 9)

b. $f(x) = -x^2 - 3x + 10$
(0, 10)

Assessment:

Question student pairs.

Independent Practice:

Text: pg. 72 – 73 prob. 2 – 20, 33A.

For a Grade:

Assignment 2.2A