**Reteach**

**9-4 Geometric Sequences and Series**

To determine whether a sequence is a geometric sequence, check for a common ratio, \( r \) \((r \neq 1)\).

Find the ratios of pairs of terms to decide whether the sequence is geometric.

| Ratios: \( \frac{6}{-2} = -3 \) | \( \frac{-18}{6} = -3 \) | \( \frac{54}{-18} = -3 \) | \( \frac{-162}{54} = -3 \) |

The common ratio is \(-3\). The sequence is geometric.

If you know the first term of a geometric sequence, \(a_1\), and the common ratio, \(r\), then you can find the \(n\)th term, \(a_n\), using the following rule.

\[ a_n = a_1 \cdot r^{n-1} \]

Find the 10th term of the geometric sequence 3, 12, 48, 192, 768, ...

**Step 1** Find the common ratio, \(r\).

\[ r = \frac{12}{3} = 4 \]

**Step 2** Identify the first term, \(a_1\).

\[ a_1 = 3 \]

**Step 3** Use the formula with \(r = 3\) to find the 10th term, \(a_{10}\).

\[ a_n = a_1 \cdot r^{n-1} \]

**Write the rule.**

\[ a_{10} = a_1 \cdot r^{10-1} \]

**Substitute** \(n = 10\).

\[ a_{10} = 3 \cdot 4^9 \]

**Substitute** \(a_1 = 3\) and \(r = 4\).

\[ a_{10} = 3 \cdot (262,144) = 786,432 \]

**Simplify.**

The 10th term of the sequence is 786,432.

**Determine whether each sequence could be geometric. If so, find the common ratio.**

1. \(-6, 12, -24, 48, -96, \ldots\)  
2. \(3, 9, 27, 81, 243, \ldots\)  
3. \(10, 60, 110, 160, 210, \ldots\)

Find the 8th term of each geometric sequence.

4. \(-7, 14, -28, 56, -112, \ldots\)  
5. \(8, 24, 72, 216, 648, \ldots\)

\[ r = \_______ \]

\[ a_1 = \_______, n = \_______ \]

\[ r = \_______ \]

\[ a_1 = \_______, n = \_______ \]
If you know any two terms in a geometric sequence, you can find any other term in the sequence.

- Find the common ratio by using the two terms and the formula for the $n$th term.
- Then use the formula for the $n$th term to find the first term and the $n$th term.

Find the 8th term of the geometric sequence with $a_4 = 162$ and $a_6 = 1458$.

**Step 1** Use the known terms and the formula for the $n$th term to find the common ratio.

\[
\begin{align*}
a_n &= a_1 r^{n-1} \\
a_6 &= a_4 r^6-4 & \text{Let } a_n = a_6 \text{ and } a_1 = a_4. \\
1458 &= 162r^2 & \text{Simplify and substitute } a_6 = 1458 \text{ and } a_4 = 162. \\
\pm 3 &= r & \text{Solve for } r.
\end{align*}
\]

**Step 2** Use one of the known terms and the common ratio, $r = \pm 3$, to find $a_1$. Use $a_4 = 162$ and the formula for the $n$th term.

\[
\begin{align*}
a_n &= a_1 r^{n-1} \\
a_4 &= a_1 (3)^{4-1} & \text{Write the formula.} \\
162 &= 27a_1 & \text{OR } a_4 = a_1 (-3)^{4-1} \\
6 &= a_1 & \text{Simplify and substitute } a_4 = 162. \\
-6 &= a_1 & \text{Solve for } a_1.
\end{align*}
\]

**Step 3** Use both cases in the formula for the $n$th term to find $a_8$.

When $r = 3$, $a_1 = 6$. When $r = -3$, $a_1 = -6$.

\[
\begin{align*}
a_n &= a_1 r^{n-1} \\
a_8 &= 6(3)^{8-1} = 13,122 & \text{In both cases, the 8th term is 13, 122.}
\end{align*}
\]

**Find the 7th term of the geometric sequence with $a_4 = 80$ and $a_5 = 160$.**

6. Find $r$.
7. Find $a_1$.
8. Find $a_7$.

\[
\begin{align*}
\text{Let } a_n = a_5 \text{ and } a_1 = a_4. & \quad \text{Let } a_n = a_4. & \quad n = \underline{\hspace{1cm}} \\
\text{Let } a_n = a_4 r^{n-1}. & \quad a_n = a_1 r^{n-1}. & \\
a_5 = a_4 r^{5-4} & \quad a_4 = a_1 (\underline{\hspace{1cm}})^{4-1} & \\
& & \underline{\hspace{1cm}} \\
& & \underline{\hspace{1cm}}
\end{align*}
\]
### 9-4 GEOMETRIC SEQUENCES AND SERIES

#### Practice A

1. a. $-384, -192, -96, -48$
   
   b. \(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\)
   
   c. Geometric; \(r = \frac{1}{2}\)

2. Geometric; \(r = 2.5\)

3. Arithmetic; \(d = -8\)

4. Geometric; \(r = -2\)

5. Neither

6. Geometric; \(r = 2\)

7. Neither

8. a. \(\frac{1}{5}\)

   b. \(\frac{1}{3125}\), or 0.00032

9. $-18.75$

10. 19,683

11. 0.0000055

12. 972

13. \(\pm 20\)

14. \(\pm 9\)

15. \(\pm 15\)

16. a. $31,026.56$

   b. $251,557.85$

#### Practice B

1. Geometric; \(r = -3\)

2. Arithmetic; \(d = 11\)

3. Neither

4. Geometric; \(r = 0.8\)

5. 3.125

6. 100,000,000,000

7. 2460.375

8. $-39,366$

9. 384

10. \(\pm 12,800\)

11. \(\pm \frac{4}{27}\)

12. \(\pm 972\)

13. \(\pm 4\)

14. \(\pm 10\)

15. \(\pm \sqrt{6}\)

16. 15,302

17. $-13,107$

18. 111,111,111

#### Practice C

1. \(\frac{1296}{625}\), or 2.0736

2. 4920.75

3. $-1088$

4. 486

5. \(\pm 139,968\)

6. \(\pm \frac{1}{135}\)

7. \(\pm 15\)

8. \(\pm 18\)

9. \(\pm 5\sqrt{3}\)

10. \(\pm 4\sqrt{17}\)

11. \(\pm 33\)

12. \(\pm 12\sqrt{2}\)

13. $-11,184,811$

14. 6187.01

15. About 31.97

16. About 52.61

17. 5555.6

18. a. $11,113.20$

   b. $41,377.20$

#### Reteach

1. Yes; \(r = -2\)

2. Yes; \(r = 3\)

3. No

4. $-2; -7; 8; a_8 = 896$

5. 3; 8; 8; \(a_8 = 17,496\)

6. 160 = 80\(r^1\); \(r = 2\)

7. 2; 80 = \(a_1(2^3)\); \(a_1 = 10\)

8. 7; \(a_7 = 10(2^6)\); \(a_7 = 640\)

#### Challenge

1. 160, 80, 40

2. 7, 10, 13

3. 2, 4, 6, 9 or \(2, \frac{1}{4}, -\frac{3}{2}, 9\)

4. When \(d = 0\), \(a_2 = 2\); When \(d = 1\), \(a_2 = 3\).

5. \(x = \frac{ac - b^2}{2b - c - a}\).

6. Possible answer: Arithmetic mean is \(\frac{a^2 + b^2}{2}\). Geometric mean is \(ab\) or \(-ab\).

   Show: \(\frac{a^2 + b^2}{2} \geq ab\) and \(\frac{a^2 + b^2}{2} \geq -ab\)

   Since all squares are nonnegative, \((a - b)^2 \geq 0\); so \(a^2 - 2ab + b^2 \geq 0\) and \(\frac{a^2 + b^2}{2} \geq ab\). Since all squares are nonnegative, \((a + b)^2 \geq 0\); so \(a^2 + 2ab + b^2 \geq 0\) and \(\frac{a^2 + b^2}{2} \geq -ab\).