

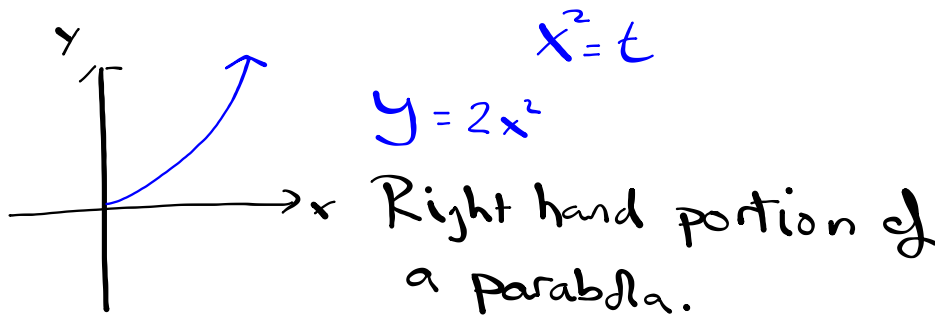


## 1.4 Parametric Equations

Sometimes it is difficult to envisage functions given rectangular  $x$  and  $y$  coordinates. When this is the case we can define  $x$  and  $y$  in terms of a 3rd parameter,  $t$ .

### Ex.1

Describe the graph determined by  $x = \sqrt{t}$  and  $y = 2t$ ,  $t \geq 0$ .



### Ex.2

Describe the graph determined by  $x = 2\cos t$  and  $y = 3\sin t$ ,  $0 \leq t \leq 2\pi$ .

$$\frac{x}{2} = \cos t \qquad \frac{y}{3} = \sin t$$

$$\frac{x^2}{4} = \cos^2 t \qquad \frac{y^2}{9} = \sin^2 t$$

$$\frac{x^2}{4} + \frac{y^2}{9} = \cancel{\cos^2 t} + \cancel{\sin^2 t}$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

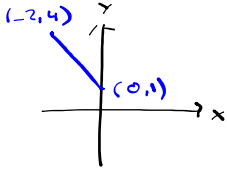
ellipse, center origin, vertically oriented.

**Ex.3**  
 Draw and identify the graph of  $x = -2t$  and  $y = 3t + 1$ ,  $0 \leq t \leq 1$ .

$\frac{x}{-2} = t$

$y = -\frac{3}{2}x + 1$

$t = 0 \quad x = 0 \quad y = 1$   
 $t = 1 \quad x = -2 \quad y = 4$



of a line segment

**TRY**  
 Draw and identify the graph of  $x = 8t - 5$  and  $y = 4t + 4$ ,  $0 \leq t \leq 1$ .

**Ex.4**  
 Find a parametrization for the line segment with end points  $(2, -3)$  and  $(1, 4)$

$x = at + b$        $y = ct + d$

assume  $0 \leq t \leq 1$

$t = 0 \quad 2 = a(0) + b \quad -3 = c(0) + d$   
 $\quad \quad \quad \underline{b = 2} \quad \quad \quad \underline{d = -3}$

$t = 1 \quad 1 = a(1) + 2 \quad 4 = c(1) - 3$   
 $\quad \quad \quad \underline{a = -1} \quad \quad \quad \underline{7 = c}$

$\underline{x = -t + 2} \quad \quad \underline{y = 7t - 3}$

**TRY**  
 Find a parametrization for the line segment with end points  $(3, 1)$  and  $(4, 5)$



## Regression Equations

Given data sets we can model the data using various types of regression equations. These equations give the best fit of an equation to the data so that predictions, extrapolations, inferences of the data can be made.

### Ex.5

The average of hourly earnings in the US, production workers for 1990-2003 are given below:

Year	Avg Hourly Earnings
1990	10.19
1991	10.50
1992	10.76
1993	11.03
1994	11.32
1995	11.64
1996	12.03
1997	12.49
1998	13.00
1999	13.47
2000	14.00
2001	14.53
2002	14.95
2003	15.35

- Produce a scatter plot of the hourly earnings as a function of years since 1990.
- Find the linear regression equation for the data and plot it on the same graph as your scatter plot.
- Use the linear regression model to predict the average hourly wage in 2020.
- Find the quadratic regression equation for the data and plot it on the same graph as your scatter plot.
- Use the quadratic regression model to predict the average hourly wage in 2020.