CALCULUS BC

# C3 Derivatives of Logs and Inverse Functions

## Log Functions

Recall that

$$y = a^u$$

$$\frac{dy}{dx} = a^u \ln a \frac{du}{dx}$$

**Ex 1**

Find the derivative of $y=3^{2x}$

$$\frac{dy}{dx} = 3^{2x} \ln(3) \cdot 2$$

$$= 2 \ln(3) 3^{2x}$$

Also $y = \log_a u$

$$\frac{dy}{dx} = \frac{1}{u \ln a} \frac{du}{dx}$$

$$\log_a u = \frac{\ln u}{\ln a}$$

$$y = \frac{1}{\ln a} \ln u$$

$$\frac{dy}{dx} = \frac{1}{\ln a} \frac{1}{u} \cdot \frac{du}{dx}$$

If you forget this formula, use the change of base formula and then take the derivative as the base will just be a constant.

**Ex 2**

Find the derivative of $y=\log_8 x^2$

$$\frac{dy}{dx} = \frac{2x}{x^2 \ln 8}$$

$$= \frac{2}{x \ln 8}$$

$$\frac{2}{x \ln 2^3}$$

$$= \frac{2}{3x \ln 2}$$

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## Logarithmic Differentiation

We use the following technique if we have a base of x and x in the exponent.

**Ex 3**

Find the derivative of $y=x^{4x}$.

$$\ln y = \ln x^{4x}$$

$$\ln y = 4x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 4\ln x + \frac{4x}{x}$$

$$\frac{1}{y}\frac{dy}{dx} = 4\ln x + 4$$

$$\frac{dy}{dx} = (4\ln x + 4)y$$

$$\frac{dy}{dx} = (4\ln x + 4)x^{4x}$$

**TRY**

Find the derivative of $y=(\cos x)^{(2x+1)}$

$$\frac{dy}{dx} = (2\ln\cos x - (2x+1)\tan x)\cos x^{2x+1}$$

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## INVERSE FUNCTIONS AND DERIVATIVES

The derivative of the inverse function is given by

3.8

$$\frac{df^{-1}}{dx}\Big|_{f(a)} = \frac{1}{\frac{df}{dx}\Big|_{a}}$$

The tangent lines of the function and its inverse have reciprocal slopes.

**Ex 4**

Given $f(x)=x^5 + 2x^3 + x - 1$ and $f(1)=3$. Find $\frac{df^{-1}}{dx}(3)$ $= \frac{1}{\frac{df}{dx}\Big|_{1}}$

$f'(x) = 5x^4 + 6x^2 + 1$

$f'(1) = 12$

$\frac{df^{-1}}{dx}\Big|_{3} = \frac{1}{12}$

**TRY**

Given $f(x)= x^3 + 3x - 2$ and $f(1)=2$. Find $\frac{df^{-1}}{dx}(2)$