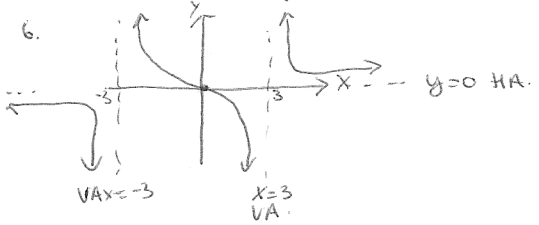
Precalculus Worksheet 2.6 - Graphing Rational Functions

1 $f(x) = \dfrac{x}{x^2-9} = \dfrac{x}{(x-3)(x+3)}$

1. Domain $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$
2. x-int $x=0$
   y-int $y=\frac{0}{0^2-9}=0$
3. HA.
   $\lim_{x\to\infty} \frac{x}{x^2} = \frac{1}{x} \to 0^+$
   $\lim_{x\to-\infty} \frac{x}{x^2} = \frac{1}{x} \to 0^-$
   Horizontal Asymptote $y=0$
4. VA.
   $x=\pm3$

   $\frac{-}{+}\ominus\ |-3|\ \oplus\ |0|\ \frac{\pm}{-}\ominus\ |3|\ \frac{\pm}{+}\oplus$
   U O U

   $\lim_{x\to-3^-} f(x) \to -\infty$ $\lim_{x\to3^-} f(x) \to -\infty$
   $\lim_{x\to-3^+} f(x) \to \infty$ $\lim_{x\to3^+} f(x) \to \infty$
5. $x=\pm3$ both infinite discontinuities
6. 

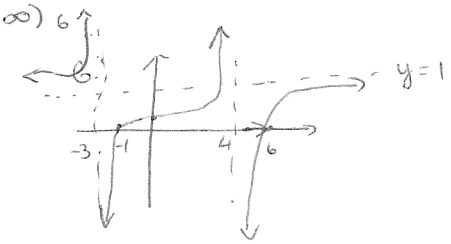
7. Range $(-\infty,\infty)$

2 $g(x) = \dfrac{x^2-5x-6}{x^2-x-12} = \dfrac{(x-6)(x+1)}{(x-4)(x+3)}$

1. Domain $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$
2. x-int $(x-6)=0$ $x=6$ $(x+1)=0$ $x=-1$
   y-int $y=\frac{0^2-5(0)-6}{0^2-0-12}=\frac{1}{2}$
3. HA: $\lim_{x\to\infty} \frac{x^2}{x^2}=1$ $y=1$
   $\lim_{x\to-\infty} \frac{x^2}{x^2}=1$
4. VA $x=-3,4$

   $\frac{(-)(-)}{(-)(-)}\oplus$ $|-3|$ $\frac{(-)(-)}{(-)(+)}\ominus$ $|-1|$ $\frac{(-)(+)}{(-)(+)}\oplus$ $|4|$ $\frac{(-)(+)}{(+)(+)}\ominus$ $|6|$
   U O U O

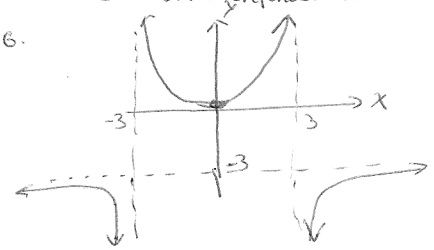
   $\lim_{x\to-3^-} g(x) \to \infty$ $\lim_{x\to4^-} g(x) \to \infty$
   $\lim_{x\to-3^+} g(x) \to -\infty$ $\lim_{x\to4^+} g(x) \to -\infty$
5. $x=-3,4$ both infinite discontinuities
7. Range $(-\infty,\infty)$



3 $h(x) = \dfrac{3x^2}{9-x^2} = \dfrac{3x^2}{(3-x)(3+x)}$

1. Domain $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$
2. x-int $x=0$
   y-int $y=0$
3. HA $\lim_{x\to\infty} \frac{3x^2}{-x^2} = -3$ $y=-3$
   $\lim_{x\to-\infty} \frac{3x^2}{-x^2} = -3$
4. VA $x=\pm3$

   $\frac{\pm}{-}\ominus\ |-3|\ \frac{\pm}{+}\oplus\ |0|\ \frac{\pm}{+}\oplus\ |3|\ \frac{\pm}{-}\ominus$
   U O U

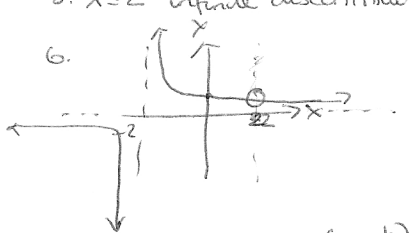
   $\lim_{x\to-3^-} h(x) \to -\infty$ $\lim_{x\to3^-} h(x) \to \infty$
   $\lim_{x\to-3^+} h(x) \to \infty$ $\lim_{x\to3^+} h(x) \to -\infty$
5. $x=\pm3$ both infinite discontinuities
6. 

7. Range $(-\infty,-3)\cup(0,\infty)$

4 $f(x) = \dfrac{2x-4}{x^2-4} = \dfrac{2(x-2)}{(x-2)(x+2)} = \dfrac{2}{(x+2)}$

1. Domain $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$
2. x-int - none
   y-int $y=1$
3. HA $\lim_{x\to\infty} \frac{2x}{x^2} = \frac{2}{x} \to 0^+$ $y=0$
   $\lim_{x\to-\infty} \frac{2x}{x^2} = \frac{2}{x} \to 0^-$
4. VA
   $x=-2$

   $\frac{\pm}{-}\ominus\ |-2|\ \frac{\pm}{+}\oplus$
   U

   $\lim_{x\to-2^-} f(x) \to -\infty$
   $\lim_{x\to-2^+} f(x) \to \infty$
5. $x=-2$ infinite discontinuity $x=2$ Removable $x=2$ $y=\frac{2}{4}=\frac{1}{2}$
6. 

7. Range $(-\infty,0)\cup(0,\frac{1}{2})\cup(\frac{1}{2},\infty)$