Graphing Power Functions

Explore
Use your calculator to compare the graphs of:

a.) $y = x^2$, $y = x^4$, $y = x^6$, $y = x^{14}$

b.) $y = x^3$, $y = x^5$, $y = x^7$, $y = x^{21}$

When the exponent is an integer, even and greater than one
The graphs look like parabolas with the vertex at $(0,0)$.

When the exponent is an integer, odd and greater than one
The graph looks like a cubic passing through the origin.
Graphing Power Functions

Explore
Use your calculator compare the graphs of

a.) $y = x^{1/2}, y = x^{1/4}, y = x^{1/6}$

b.) $y = x^{1/3}, y = x^{1/5}, y = x^{1/7}$

When the exponent is a fraction, even and less than one
The graph looks like a square root function with the Extrema at the Origin.

When the exponent is a fraction, odd and less than one
The graph looks like a cube root function passing through the Origin.
If the index outside the radical is even are you allowed negatives values? What if the index is odd?

\[ b \sqrt[\text{index}]{\frac{a}{\text{power}}} \]

<table>
<thead>
<tr>
<th>Even index</th>
<th>Odd index</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Even power → Yes</td>
<td>Odd power → No</td>
</tr>
</tbody>
</table>

**Ex.2**

State the values of the constants a and k. Sketch the graph by hand, using a table of values if necessary. It might also help to think about the domains.

\[ a \]

\[ k = 1 \]

\[ f(x) = \sqrt[3]{x} \]

\[ g(x) = \sqrt[3]{x^2} \]

\[ a = \frac{2}{3} \]

\[ k = 1 \]

*Reflect the ---- because squaring made it positive.*
Powers >1

Ex. 3
State the values of the constants a and k. Sketch the graph by hand, using a table of values if necessary.

a) \( f(x) = \frac{4}{5} \sqrt[5]{x^4} \)
\( f(x) = 1 \cdot x \frac{4}{5} \)

\( a = \frac{5}{4} \)
\( k = 1 \)

b) \( f(x) = \frac{3}{4} \sqrt[4]{x^3} \)
\( f(x) = 1 \cdot x^{\frac{3}{4}} \)

\( a = \frac{4}{3} \)
\( k = 1 \)

Don't include left side because we can't take \( \sqrt{ } \) of a negative.
Sketch a graph of:

\[ f(x) = 3 \sqrt[12]{x^8} \]
\[ f(x) = x \]
\[ \frac{\text{even}}{\text{even}} > 1 \]

\[ f(x) = 13 \sqrt[7]{x} \]
\[ f(x) = x \]
\[ \frac{\text{odd}}{\text{even}} < 1 \]
**Fractional Functions**

\[ a < 0 \]

**Explore**

Use your calculator to compare the graphs of:

a.) \( y = x^{-2}, y = x^{-4}, y = x^{-6} \)

b.) \( y = x^{-3}, y = x^{-5}, y = x^{-7} \)

**Ex. 4**

State the values of the constants \( a \) and \( k \). Sketch the graph by hand, using a table of values if necessary.

a) \( f(x) = \frac{1}{x^{4/5}} \)

\[ f(x) = x^{-4/5} \]

\[ a = -\frac{4}{5} \]

\[ k = 1 \]

b) \( f(x) = \frac{1}{x^{3/7}} \)

\[ f(x) = x^{-3/7} \]

\[ a = -\frac{3}{7} \]

\[ k = 1 \]
For $a > 1$:

- $\frac{4}{2}x^2$
- $\frac{2}{3}x^3$
- $\frac{5}{2}x^\frac{5}{2}$

*Follow numerator*

*Use denominator for the left hand side.*
0 < a < 1

*follows denominator.

& if even numerator.
$a < 0$

- Even
- Odd
- Odd
- Even
In General...

*Know the right hand side.

\[ a > 1 \quad 0 < a < 1 \quad a < 0 \]

\[
\begin{array}{ccc}
\uparrow & & \uparrow \\
\rightarrow & & \rightarrow
\end{array}
\]

*Determine left hand side using signs.

1. Doesn't exist

\[
\begin{array}{ccc}
\uparrow & & \uparrow \\
\rightarrow & & \rightarrow
\end{array}
\]

2. Is positive

\[
\begin{array}{ccc}
\uparrow & & \uparrow \\
\leftarrow & & \leftarrow
\end{array}
\]

3. Is negative

\[
\begin{array}{ccc}
\downarrow & & \downarrow \\
\rightarrow & & \rightarrow
\end{array}
\]