Projectile Motion: Solving Problems With Angles

Ch. 5 in your text book

Students will be able to:
1) Calculate the horizontal and vertical velocity components of a velocity vector
2) Solve projectile motion problems involving angles
Velocity Vector Components

We saw this diagram previously showing the horizontal and vertical components to a velocity vector.

Usually, you are told the velocity of an object and the angle at which it was initially moving. To answer questions, you’ll need to find the horizontal and vertical components to that velocity.
Velocity Vector Components

To find these components, we will be using the math functions of sine and cosine. Don’t worry if you haven’t got to this in math yet – we’ll show you what to do.

\[
\sin(\theta) = \frac{opp}{hyp}
\]

\[
\cos(\theta) = \frac{adj}{hyp}
\]

\(\theta\) is the variable for an angle.
Velocity Vector Components

Pull out your calculators and make sure you are in “degree” mode.

Example: Find \( \sin(30) \) and \( \cos(30) \).

\[
\sin(\theta) = \frac{\text{opp}}{\text{hyp}} \quad \cos(\theta) = \frac{\text{adj}}{\text{hyp}}
\]

\[
\sin(30) = \frac{1}{2} \quad \cos(30) = \frac{\sqrt{3}}{2}
\]
Velocity Vector Components

\[
\sin(\theta) = \frac{opp}{hyp} \quad \cos(\theta) = \frac{adj}{hyp}
\]

\[
\sin(\theta) = \frac{v_v}{v} \quad \cos(\theta) = \frac{v_h}{v}
\]

\(\theta\) is the variable for an angle.

Each side of the triangle represents something about the object’s velocity.
Velocity Vector Components

\[ v_v = vsin(\theta) \]
\[ v_h = vcos(\theta) \]

\[ \theta \] is the variable for an angle

*make sure you are in “degree” mode
Velocity Vector Components

Example: An object is kicked at a velocity of 2 m/s at an angle of 30°. Find the horizontal and vertical velocity components.

\[ v_v = v \sin(\theta) \quad v_h = v \cos(\theta) \]
\[ v_v = (2) \sin(30) \quad v_h = (2) \cos(30) \]
\[ v_v = (2)(.5) \quad v_h = (2)(.866) \]

\[ v_v = 1 \text{ m/s} \quad v_h = 1.73 \text{ m/s} \]

*make sure you are in “degree” mode
Velocity Vector Components

Check for understanding: An football is punted with an initial velocity of 12 m/s at an angle of 65°. Find the horizontal and vertical velocity components.

\[ v_v = v \sin(\theta) \]
\[ v_h = v \cos(\theta) \]

\[ v_v = (12) \sin(65) = 10.88 \text{ m/s} \]
\[ v_h = (12) \cos(65) = 5.07 \text{ m/s} \]

*make sure you are in “degree” mode
Velocity Vector Components

Check for understanding: A baseball is hit with an initial velocity of 49 m/s at an angle of 30°. How long will the ball be in the air?

\[ \nu_v = v \sin(\theta) \quad \nu_h = v \cos(\theta) \]

\[ = (49) \sin(30) \quad = (49) \cos(30) \]

\[ = 24.5 \text{ m/s} \quad = 42.44 \text{ m/s} \]

*make sure you are in “degree” mode
Solving Problems
Keep the vertical information separate from the horizontal information.

<table>
<thead>
<tr>
<th>Vertical Information</th>
<th>Horizontal Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_i = 24.5 \text{ m/s})</td>
<td>(v = 42.44 \text{ m/s})</td>
</tr>
<tr>
<td>(v_f = -24.5 \text{ m/s})</td>
<td>(x = ?)</td>
</tr>
<tr>
<td>(a = -10 \text{ m/s}^2)</td>
<td>(t = ?)</td>
</tr>
<tr>
<td>(x = 0)</td>
<td>(t = 4.9 \text{ s})</td>
</tr>
<tr>
<td>(t = ?)</td>
<td>(v_f = at + v_i)</td>
</tr>
</tbody>
</table>

\[x = v_i t + \frac{at^2}{2}\]
\[0 = 24.5t + \frac{(-10)(t)^2}{2}\]
\[t = 4.9 \text{ s}\]
\[-24.5 = -10t + 24.5\]
\[t = 4.9 \text{ s}\]
Additional Angle Information

When something is shot/kicked/thrown horizontally, the angle is $0^\circ$.

$v_v = v \sin(\theta)$  \hspace{1cm} $v_h = v \cos(\theta)$

$= (10) \sin(0)$ \hspace{1cm} $= (10) \cos(0)$

$= 0 \, m/s$ \hspace{1cm} $= 10 \, m/s$

$v = 10 \, m/s$

$v_v = 0 \, m/s$

$v_h = 10 \, m/s$

*make sure you are in “degree” mode*
Additional Angle Information

When something is shot/kicked/thrown vertically, the angle is 90°.

$v = 10 \text{ m/s}$

$v_v = 10 \text{ m/s}

v_h = 0 \text{ m/s}

$v = v_\sin(\theta) = (10)\sin(90) = (10)\cos(90) = 10 \text{ m/s}

*make sure you are in “degree” mode
Solving Problems

Key concepts to remember:

When something is dropped, it’s initial **vertical** velocity is 0

When something is thrown **horizontally**, it’s initial **vertical** velocity is 0

For the **vertical** direction, the acceleration will always be due to gravity (-9.81 m/s² on Earth)

At it’s highest point, the **vertical** velocity of an object is 0
Solving Problems

Key concepts to remember:

Velocities and displacements are vectors; they can be positive or negative.

The initial and final velocities for something that goes up and down will be in opposite directions but have the same magnitude.

The time it takes an object to go up and down is twice as long as it takes to go up.

The horizontal velocity won’t change while the object is in flight.
Solving Problems

Constant acceleration equations:
\[ x = v_i t + \frac{1}{2} a t^2 \]
\[ 2a x = v_f^2 - v_i^2 \]
\[ v_f = at + v_i \]

Constant velocity equation:
\[ x = vt \]

Which of these variable does not have a direction?

Time is what links the two directions together. It will be the same in all the equations for a given problem.
Example: A ball is shot horizontally at 3 m/s and is in the air for 3 seconds. How far did it go?

What key words are there?

Are they asking about horizontal or vertical information?

\[ x = vt \]
\[ x = (3)(3) \]
\[ x = 9\text{m} \]
Solving Problems

Example: A ball is shot horizontally at 3 m/s and is in the air for 3 seconds. How far did the ball fall in that time?

What key words are there?

Are they asking about horizontal or vertical information?

Given \textit{vertical} information:

\begin{align*}
  v_i &= 0 \\
  a &= -10 \text{ m/s}^2 \\
  t &= 3 \text{ s}
\end{align*}

\begin{align*}
  x &= v_i t + \frac{at^2}{2} \\
  &= 0 + \frac{(-10)(3)^2}{2} \\
  &= -45 \text{ m}
\end{align*}
Solving Problems

Example: A ball is thrown up at an angle. The vertical velocity is initially 2 m/s. It reaches its highest point after .2 seconds. How high did it go?

What key words are there?

Are they asking about horizontal or vertical information?
Example: A ball is shot from the ground at an angle. The vertical velocity was initially 5 m/s. How long did it take before it came back to the ground?

What key words are there?

Are they asking about horizontal or vertical information?