1.4 Building Functions from Functions

Let $f(x)=x^2$ and $g(x)=\sqrt{x+1}$

D: $(-\infty,\infty)$ $[-1,\infty)$

Find the following and give the domain of each new function.

Domain

$f+g = x^2+\sqrt{x+1}$ $[-1,\infty)$

$f-g = x^2-\sqrt{x+1}$ $[-1,\infty)$

$f/g = \frac{x^2}{\sqrt{x+1}}$ $(-1,\infty)$

$g/f = \frac{\sqrt{x+1}}{x^2}$ $[-1,0)\cup(0,\infty)$

$gg = \sqrt{x+1}\sqrt{x+1}$ $[-1,\infty)$

$= (x+1)$

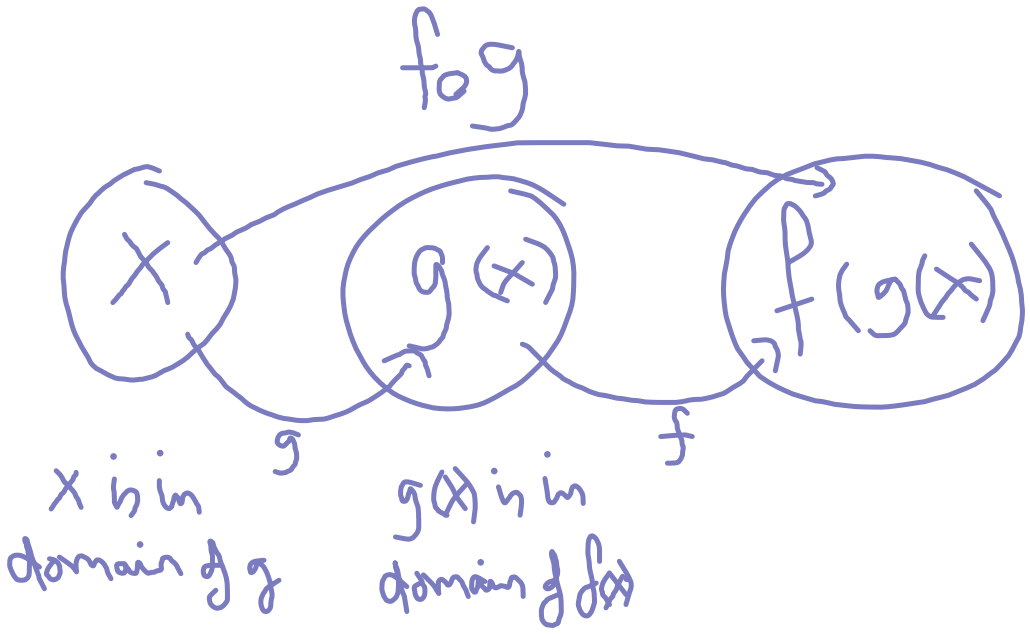
PRECALC

When combining functions, the composition $f \circ g$ is denoted:

$$(f \circ g)(x) = f(g(x))$$

$= f \circ g$

The domain of $f \circ g$ consists of all x-values in the domain of g that map to $g(x)$ values in the domain of f.



Ex.1 Let f(x)=2^x and g(x)=$\sqrt{x}$. Find $f \circ g(x)$ and $g \circ f(x)$ and. Verify the functions $f \circ g$ and $g \circ f$ are not the same.

$f(x) = 2^x$ D: $(-\infty, \infty)$ $g(x) = \sqrt{x}$ D: $[0, \infty)$

$f \circ g(x)$
$= f(g(x))$
$= f(\sqrt{x}) = 2^{\sqrt{x}}$ $[0, \infty)$

$g \circ f(x) = g(f(x))$
$= g(2^x)$
$= \sqrt{2^x}$ $(-\infty, \infty)$

The domain of $f \circ g$ is defined for $[0, \infty)$.
The domain of $g \circ f$ is defined for $(-\infty, \infty)$.

We could also verify graphically.

Try Repeat Ex 1 for

$f(x) = 2x^2$ $(-\infty, \infty)$ $g(x) = \frac{1}{x}$ $(-\infty, 0) \cup (0, \infty)$

$f \circ g(x) = 2\left(\frac{1}{x}\right)^2$
$= \frac{2}{x^2}$
$(-\infty, 0) \cup (0, \infty)$

$g \circ f(x) = \frac{1}{2x^2}$
$(-\infty, 0) \cup (0, \infty)$

Ex.2

Find $f \circ g(2)$ and $g \circ f(-3)$ given $f(x)=x^2+2x-1$ and $g(x)=x-4$

$f \circ g(x)$
$= f(g(x)) = f(x-4)$
$= f(g(2))$
$= f(2-4)$
$= f(-2)$
$= (-2)^2+2(-2)-1$
$= 4-4-1$
$= -1$

$g \circ f(x)$
$= g(f(x))$
$g(f(-3))$
$f(-3) = (-3)^2+2(-3)-1$
$= 9-6-1$
$= +2$

$g \circ f(-3)$ $g(2) = 2-4$
$= -2$

TRY

Find $g \circ f(4)$ *and* $f \circ g(-2)$
given $f(x)=\sqrt{x}-3$ and $g(x)=3x^2-12$

$g \circ f(4)$
$= g(f(4))$
$f(4) = \sqrt{4}-3$
$= -1$
$g(-1) = -9$

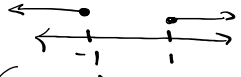
$f \circ g(-2)$
$f(g(-2))$
$g(-2) = 0$
$f(0) = -3$

Ex.3

Find the domain of the composite functions $g \circ f$ *and* $f \circ g$
given $f(x)=x^2-1$ and $g(x)=\sqrt{x}$

$[0,\infty)$

$(-\infty,\infty)$
$g \circ f = g(f(x))$
$= \sqrt{x^2-1}$
$x^2-1 \geq 0$
$x^2 \geq 1$
$x \geq 1$ or $x \leq -1$



$(-\infty,-1) \cup (1,\infty)$

$f \circ g = f(g(x)) = (\sqrt{x})^2 - 1$
$= x-1$
$[0,\infty)$

TRY

Find the domain of the composite functions $g \circ f$ *and* $f \circ g$
given $f(x)= 1/(x-1)$ and $g(x)=\sqrt{x}$.

Decomposing Functions

Ex.4 If $h(x)=\frac{1}{x^2-9}+4(x^2-9)^3$ find functions f(x) and g(x) such that h(x)=f(g(x)).

Let $g(x)=x^2-9$

$f(x)=\frac{1}{x}+4x^3$

or Let $g(x)=x^2$

$f(x)=\frac{1}{x-9}+4(x-9)^3$

TRY If $h(x)=-2(x^2-1)^3+x^2-1$ find functions f(x) and g(x) such that h(x)=f(g(x)).

x x

Let $g(x)=x^2-1$

$f(x)=-2x^3+x$

$-2(x^2-1)^3+x^2-1$

Try Repeat for $h(x)=e^{\cos x}$

Let $g(x)=\cos x \rightarrow x$

$f(x)=e^x$

$h(x)=f(g(x))$

$g(f(x))$

$f(x)=\cos x$

$g(x)=e^x$

Try Repeat $h(x)=f(g(x))$

$h(x)=3|2x-4|+1$

Let $g(x)=|2x-4| \rightarrow x$

$f(x)=3x+1$

or

Let $g(x)=2x-4$

$f(x)=3|x|+1$

Using Implicitly Defined Functions

Ex.5

Describe the graph of the relation $9x^2 - 12xy + 4y^2 = 16$.

$9x^2 - 12xy + 4y^2 = 16$

$(3x - 2y)^2 = 16$

$2(3x)(2y)$

$(3x-2y)^2 = 16$

$(3x-2y) = \pm 4$

$3x-2y=4$ or $3x-2y=-4$

$-2y = -3x+4$ or $-2y = -3x-4$

$y = \frac{3}{2}x-2$ $\quad y = \frac{3}{2}x+2$

Two parallel lines.

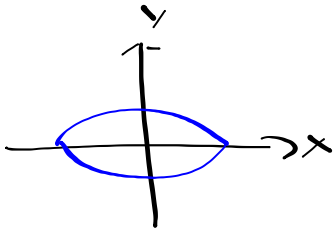
Ex.6

Describe the graph of the relation $x^2 + 7y^2=84$.

$7y^2 = -x^2+84$

$y^2 = -\frac{1}{7}x^2+12$

$y = \pm\sqrt{-\frac{1}{7}x^2+12}$



Ex.7

Describe the graph of the relation $x+3|y|=9$

$3|y| = -x+9$

$|y| = -\frac{1}{3}x+3$

$y = -\frac{1}{3}x+3$ $\quad y = -(-\frac{1}{3}x+3)$

$y = \frac{1}{3}x-3$

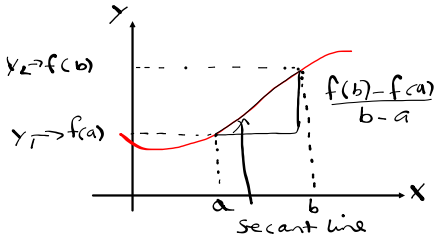
Two intersecting lines.

Average Rate of Change of Functions

The average rate of change of a function is defined by:

$m = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} =$

Avg rate of change = $\frac{f(b) - f(a)}{b - a}$



So the average rate of change is the slope of the SECANT line between x = a and x = b on the graph that passes through the points (a,f(a))and (b,f(b)).

Ex.8

Find the average rate of change of the for the function $f(x) = x^2 + 4$ between the following sets of points:

a) x = 2 and x = 6
b) x = 5 and x = 10
c) x = a and x = a + h

$f(b) =$
$f(6) = 6^2 + 4 = 40$
$f(2) = 2^2 + 4 = 8$

a) $\text{Avg ROC} = \frac{f(b) - f(a)}{b - a}$

$= \frac{40 - 8}{6 - 2}$

$= \frac{32}{4}$

$= 8$

b) $\text{Avg ROC} = \frac{f(10) - f(5)}{10 - 5}$

$= \frac{104 - 29}{5}$

$= 15$

$f(x) = x^2 + 4$

c) $\text{Avg ROC} = \frac{f(a+h) - f(a)}{a+h-a}$

$= \frac{(a+h)^2 + 4 - [(a)^2 + 4]}{h}$

$(a+h)(a+h)$
$a^2 + ah + ha + h^2$
$a^2 + 2ah + h^2$

$= \frac{a^2 + 2ah + h^2 + 4 - a^2 - 4}{h}$

$= \frac{2ah + h^2}{h}$

$= \frac{2ah}{h} + \frac{h^2}{h}$

$= 2a + h$

Ex.9

Given g(x)= -7x + 2, and f(x)=2x + 3 find the average rate of change of each function between x = a and x = a + h. Discuss with your neighbor what you notice about the average rate of change for each function and the two given equations.

$\text{Avg ROC}_{g(x)} = \frac{-7(a+h) + 2 - (-7a + 2)}{a+h-a}$

$= \frac{-7a - 7h + 2 + 7a - 2}{h}$

$= \frac{-7h}{h}$

$= -7$

$\text{Avg ROC}_{f(x)} = 2x + 3$

$\frac{2(a+h) + 3 - (2a + 3)}{a+h-a}$

$= \frac{2a + 2h + 3 - 2a - 3}{h}$

$= \frac{2h}{h}$

$= 2$